HYBRID MONTE CARLO FORM APPROACH TO DETERMINE ACCEPTABLE DEFECT SIZE FOR RIGID WELDED RISERS

Wellison José de Santana Gomes[1]

Alexandre Galiani Garmbis[2]

André Teófilo Beck[3]

ABSTRACT

Managing the risk of operating welded rigid risers requires tools for establishing acceptable defect sizes. In a reliability-based approach, these are determined based on a target reliability, which needs to be respected for the lifetime of the riser. This is an inverse fracture mechanics problem, since one needs to find the acceptable defect size at time zero, which leads to an acceptable failure probability at the end of the design life. Also, one needs to consider random sea states, random crack propagation parameters, uncertain initial crack sizes and their probabilities of detection. In order to solve the inverse stochastic fracture mechanics problem, we propose a hybrid method, which combines the best features of Monte Carlo Simulation (MCS) and the First Order Reliability Method (FORM). The non-linear crack propagation phase of the problem is handled by MCS; the final fracture problem is handled by FORM; and the allowable crack size, required in order to impose a minimum lifetime reliability, is obtained by simple root-finding, among the set of initial crack size samples. Efficiency is achieved: a) by classifying initial crack size samples, and computing only those terms effectively contributing non-zero probabilities to the integral; and b) by solving the optimization problem using the information acquired from a single Monte Carlo run. The proposed hybrid approach is employed in solution of typical welded riser crack propagation problems: it is shown that it gains efficiency when the target reliability is large, as expected in practical structural engineering problems.

1. INTRODUCTION

The inverse fracture mechanics problem to be addressed herein is a particular case of inverse reliability problem. In inverse structural reliability problems, a target reliability level is established, and one looks for the structural parameters for which required reliability levels are achieved [1, 2]. Following Mínguez, Castillo and Hadi [3], these problems may be seen as special cases of the more general Reliability-Based Design Optimization (RBDO) problems [4-9].

In the literature, many methods have been proposed to solve inverse reliability problems. The first attempts to deal with these problems [1-2, 10-13] basically employed transformations and algorithms associated with the First Order Reliability Method (FORM), such as the transformation to the normal standard space and some modified versions of the HLRF (Hasofer-Lind-Rackwitz-Fiessler) algorithm. Although the applications are broad, practical examples related to the offshore industry appear in many of these papers, e.g. [2, 11-12].

Unfortunately, for certain problems, most of the methods cited above lose efficiency, or are not applicable at all. This happens, for example, in problems presenting complex time-dependency (e.g. non-linearity), or multi-dimensional problems, like those involving metal fatigue. In the latter case, depending on how the crack propagation phenomena is modelled, large numbers of load cycles may lead to very high problem-dimensionality, with thousands to millions of variables, and gradients which are either unstable by nature and/or difficult to compute. Most of the methods cited above depend on gradient calculations. Moreover, use of surrogate models, which could help to alleviate the computational burden, is often inefficient for high

1 D.Sc., Civil Engineer, Federal University of Santa Catarina.
2 M.Eng., Mechanical Engineer, PETROBRAS
3 Ph.D., Mechanical Engineer, University of São Paulo.

dimensional problems. Adding to the difficulties above, metal fatigue is significantly impacted by uncertainties in stochastic loads and in random crack growth parameters. Hence, the necessity to develop specific methods to solve inverse time-variant reliability (fatigue) problems should be evident.

The problem of our immediate interest relates to acceptance criteria for flaws in metallic structures. More specifically, interest lies in establishing the maximum size of initial flaws, such that an offshore structure subject to millions of random load cycles and to significant uncertainty in crack growth parameters, reaches its intended design life with a pre-determined reliability level. Solution to this problem can be very challenging, as it involves a very large numbers of random variables and/or stochastic processes, unstable gradients and random variables with strongly non-Gaussian distributions. This includes truncated flaw size distributions, which may cause problems in pure FORM-based algorithms. The proposed approach targets high efficient and stability, as a trade-off for the greater generality of other methods from the literature.

2. STRUCTURAL RELIABILITY, FRACTURE MECHANICS AND THE INVERSE RELIABILITY PROBLEM

2.1 Structural reliability

Let us consider n_{rv} random variables grouped into a vector $\boldsymbol{X} = \{X_1, X_2, \ldots, X_{n_{rv}}\}$, with joint probability density function (PDF) $f_{\boldsymbol{X}}(\boldsymbol{x})$, and whose realizations are represented by $\boldsymbol{x} = \{x_1, x_2, \ldots, x_{n_{rv}}\}$. In the time-dependent problems considered herein, the vector of random variables may also represent stochastic processes, such as the fatigue crack growth, which are discretized over a number of load cycles. In this case, correlation between some random variables could be necessary to correctly describe the stochastic processes at hand.

Failure is characterized by a limit state function, $g(\boldsymbol{X})$, which divides the failure and safe domains, Ω_f and Ω_s, given by:

$$\begin{aligned} \Omega_f &= \{\boldsymbol{x} | g(\boldsymbol{x}) \leq 0\} \\ \Omega_s &= \{\boldsymbol{x} | g(\boldsymbol{x}) > 0\} \end{aligned} \tag{1}$$

so that $g(\boldsymbol{x}) \leq 0$ indicates failure of the structure for a given realization, $\boldsymbol{x}$, of the vector of random variables. The limit state function can be a single analytical function, but can also involve more complex analytical and/or numerical terms.

By definition, the failure probability related to $g(\boldsymbol{x})$ is given by:

$$P_f = P\left[\boldsymbol{X} \in \Omega_f\right] = \int_{\Omega_f} f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x}. \tag{2}$$

The failure probability is associated to the so-called reliability index, β, by means of:

$$P_f = \Phi(-\beta), \tag{3}$$

where $\Phi(.)$ is the cumulative distribution function (CDF) of the standard normal distribution.

Only in a few cases the multidimensional integral in Eq. (2) can be solved analytically, or by means of classical numerical integration. In most cases, solutions are only possible or viable via specific structural reliability methods, such as FORM and MCS. These methods are described, for example, in Ditlevsen and Madsen [38] and Melchers and Beck [39].

Solution by simple, or crude, MCS is obtained via Eq. (4), by randomly generating n_{MC} samples of $\boldsymbol{X}$ according to its joint distribution, $f_{\boldsymbol{X}}(\boldsymbol{x})$, and evaluating a so-called indicator function, $I[\boldsymbol{x}]$, on these samples. For each sample $\boldsymbol{x}_i$, which corresponds to the ith simulation, the indicator function results one if $\boldsymbol{x}_i$ belongs to the failure domain, and zero otherwise.

$$P_f = E\big[I[\boldsymbol{X}]\big] \cong \frac{1}{n_{MC}} \sum_{i=1}^{n_{MC}} I[\boldsymbol{x}_i]. \tag{4}$$

In general, the smaller the structural failure probability, the higher the number of samples required to achieve convergence of the failure probability estimate given by Eq. (4). This may lead to very large computational costs, depending on the computational expense required per simulation, especially because engineering structures usually present very small failure probabilities. However, although many simulation-based methods with faster convergence are available in the literature [40-42], crude MCS is a very simple and general method, employed in the solution of many problems and commonly taken as a reference to verify the accuracy of other methods.

On the other hand, solution by FORM requires transformation of the problem to standard normal space, where all random variables are normally distributed, with zero means and unitary standard deviations. This transformation is non-linear for correlated random variables, and for highly non-Gaussian probability distributions [43]. Within the standard normal space, an iterative procedure is employed to find the point of maximum likelihood among those over the limit state equation, $g(\boldsymbol{x}) = 0$, usually known as the most probable failure point (MPP). The limit state equation is linearized at this point, and, as the reliability index corresponds to the distance between the MPP and the origin of the standard normal space, the failure probability is computed using Eq. (3). The search for the MPP is often performed by means of specialized algorithms, such as the improved HLRF method [10], and using convergence criteria based on the limit state value and on orthogonality. In this paper, FORM solutions are obtained using the limit state value and orthogonality convergence criteria, both with a tolerance of 10^{-8}. The iterative procedure, as well as the check for convergence, depends on the computation of limit state function gradients. Thus, FORM is not indicated for problems where the linearization leads to large errors, and it may be difficult to apply in cases where the computation of gradients is challenging.

2.2 Fracture mechanics

Metallic structures under cyclic loading are subject to the development and propagation of cracks, a phenomenon known as fatigue, which may lead to catastrophic structural failures [44-45]. Nonmetals are not immune to fatigue as well, as stated by Anderson [46].

In structural reliability problems involving crack propagation, as in many other time-dependent reliability problems, large uncertainties are present. Uncertainty and uncertain variables may be found at different stages of a structure's life: at initial time (t_0), at the propagation phase, and at the final time, or design life (t_D). Stochastic processes are usually present in the propagation phase; this can be the case in fracture mechanics, as shown by Beck and Melchers [47].

At the initial time, one or more crack geometry parameters may be considered uncertain, and represented by random variables, functions of random variables, random fields or other mathematical models. The initial crack depth, a_0, for example, is often taken as an important random variable. Even before cyclic loading starts, the probability distributions describing initial crack size may change, due to the application of inspections and/or interventions. These changes could also take into account uncertainties related to the inspections and interventions, by means, for example, of probability of detection curves and uncertainties in measurements [48-51]. One type of intervention, with potentially significant impact in the probability distributions of initial crack depth, is the application of acceptance criteria, followed by possible replacements or repair of major flaws. Use of an acceptance criterion on initial flaw size is a way to control the reliability of the structural system or component, so that it achieves pre-specified reliability levels.

Without loss of generality, and in benefit of explanation clarity, let us consider the initial crack depth, with probability density function $f_{a_0}(a_0)$, as the only random variable, at initial time. With focus on the initial crack depth, a simple acceptance criterion, based on an acceptable initial crack depth, a_{ac}, is considered. The structure or component is rejected and replaced, in case $a_0 \geq a_{ac}$, and accepted otherwise. The acceptance criterion has the effect of truncating the probability density $f_{a_0}(a_0)$, leading to a conditional distribution of a_0 given a_{ac}, $f_{a_0|a_{ac}}(a_0)$.

The scenario where no acceptance criteria are applied is investigated first. In this case, the initial crack has a probability distribution described by $f_{a_0}(a_0)$, and the structure is submitted to a high number of load cycles, along its lifetime, giving rise to the propagation phase.

In the propagation phase, crack depth is driven by stochastic load processes, and depends on crack growth parameters, which can be described as random variables or stochastic processes [47]. Significant uncertainty is added to the problem at the propagation phase, which encompasses the lifespan of the structure; hence, crack depth a, is represented as a stochastic process of time, $a(t)$, or as function of the discrete number of load cycles $a(N)$. Again, note that more complex descriptions of crack growth over time could be used.

The crack growth process may be described by different models [46, 50-52]. Among them, the Paris' law [46, 52-53] has found many applications in the literature, and is adopted herein:

$$\frac{da}{dN} = C\Delta K^m, \tag{5}$$

where $\frac{da}{dN}$ is the differential crack growth per cycle, ΔK is the range of stress-intensity factor within the cycle, and C and m are material constants that are determined experimentally. A fatigue threshold, ΔK_{th}, may be considered, so that for $\Delta K < \Delta K_{th}$ no growth occurs, as observed experimentally [46].

The crack depth for a given number of loading cycles, N, which corresponds to a given time t, is obtained by integrating Paris' law over the number of cycles or over the time interval [t_0, t]:

$$a(N) = a_0 + \int_1^N C\Delta K^m dN; \tag{6a}$$

$$a(t) = a_0 + \int_{t_0}^t C\Delta K^m \frac{dN}{dt} dt. \tag{6b}$$

Stress-intensity factors, K, may be determined by means of numerical methods, such as the finite element method. Results obtained via numerical models are directly used in lifecycle analyses, summarized in tables or employed in the construction of approximations [54-56], such as those based on polynomial functions, which are later employed in the analyses. Computation of maximum and minimum values of K over a load cycle allows determination of the respective range ΔK. The range, ΔK, is usually written as a function of crack depth, in the form:

$$\Delta K(a) = Y(a)\Delta\sigma\sqrt{\frac{\pi a}{Q}}, \tag{7}$$

where $\Delta\sigma$ is the stress range over the cycle and $Y(\ )$ is the geometry factor. The empirical expressions commonly used for Q for nearly semi-elliptical surface cracks in pipes are [57]:

$$Q = \begin{cases} 1 + 1.464(a/c)^{1.65}, for\ a/c \leq 1 \\ 1 + 1.464(c/a)^{1.65}, for\ a/c > 1' \end{cases} \tag{8}$$

Considering Eq. (7), and noting that the integral in Eq. (6) involve a discrete number of terms (cycles), Eq. (6) may be rewritten as:

$$a(N) = a_0 + \sum_{i=1}^{N} C\left(Y(a_{i-1})\Delta\sigma_i\sqrt{\frac{\pi a_{i-1}}{Q}}\right)^m, \tag{9}$$

where it is highlighted that $\Delta\sigma$ may vary from cycle to cycle (for instance, following a time series), and that each value a_i depends on the previous value, a_{i-1}. The crack depth at the end of the lifetime, a_f, is assessed by evaluating Eq. (9) for the total number of load cycles, N_D, corresponding to the end of a design life, t_D.

Note that uncertainties related to the initial crack depth propagate during the lifetime, and are affected by the uncertainties related to crack growth and to the load effects, here represented by ΔK. All these uncertainties contribute to the uncertainties of final crack depth; hence, also contribute to the possibility of undesirable responses (failure).

When an initial or fabrication acceptance criterion is employed, inspection and rejection of large flaws truncate the probability distribution of initial crack depths, with direct impact in the probability distributions at the final time. The initial and final crack depths have distributions given by $f_{a_0}(a_0)$ and $f_{a_f}(a_f)$, respectively, if no acceptance criteria are applied. Upon application of the previously described criterion, they become conditionally distributed, with PDFs $f_{a_0|a_{ac}}(a_0)$ and $f_{a_f|a_{ac}}(a_f)$, respectively.

At any instant along the lifetime of the structure, with acceptance criteria applied or not, reliability analysis may be performed by considering one or more limit state functions, and using appropriate time-dependent reliability procedures, to determine if the structure achieves/maintains desired reliability levels. In many problems, inspections and interventions may be applied several times along the lifespan, to help achieving the required reliability levels.

If undetected, a fatigue crack may propagate until it reaches a critical crack size, that is, a crack size that produces a stress intensity factor that exceeds the limiting value for stable propagation. At this stage, crack propagation becomes unstable and final rupture occurs. The limit state function can be written in terms of a critical crack size, a_{cr}, which is related to the critical value of the stress intensity factor, $K(a_{cr}) = K_{cr}$, which leads to unstable growth and rupture:

$$\begin{aligned} g_n(\boldsymbol{X}) &= R(\boldsymbol{X}^{frac}) - S(\boldsymbol{X}^{prop}) \\ &= a_{cr} - a_n(a_0, \boldsymbol{a}_i, C, m, \Delta\sigma). \end{aligned} \tag{10}$$

In Eq. (10), sub-index $()_n$ is the accumulated cycle counter, which denotes the problem's time-dependency; $R(\quad)$ is a resistance (strength) function; $S(\quad)$ is a load effect function. In order to describe the proposed hybrid method, it is convenient to decompose the random variable vector $\boldsymbol{X}$ as follows: $\boldsymbol{X} = \{\boldsymbol{X}^{prop}, \boldsymbol{X}^{frac}\}$; where the crack propagation variables are grouped in $\boldsymbol{X}^{prop} = \{a_0, \boldsymbol{a}_i, C, m, \Delta\sigma, \ldots\}$, where $\boldsymbol{a}_i, (i = 1, \ldots, n)$ represents the vector of random crack depths obtained over the lifetime; and the final strength variables are grouped in $\boldsymbol{X}^{frac} = \{a_{cr}, K_{cr}, \sigma_y, \ldots\}$. Note that the first line in Eq. (10) targets the general case, where the strength function $R(\quad)$ can be, for instance, an elastic-plastic Failure Assessment Diagram [46]. The second line states the more simple case considered herein, for demonstration purposes, where strength is given simply by a_{cr} (elastic fracture). In the general case, Eq. (10) can be evaluated for any number of cycles n, up to desing live N_D. In this paper, a minimum reliability level is imposed at the end of design live; hence, Eq. (10) is evaluated only for $n = N_D$.

2.3 Inverse reliability problem

When a structure or component is subject to stochastic processes along its lifespan, it may be necessary to verify, at the fabrication stage, if the structure or component can achieve the proposed design life, with a specified reliability level. It may be required, for instance, that the reliability index at the end of lifetime, $\beta(t_D)$, must be greater than a specified target, β_T.

In case of metal structures or components subject to cyclic loading, and which are produced by welding, it is often assumed that welding introduces small flaws, which may grow to critical size during the lifetime. Some structures can be inspected and repaired over time, in order to keep their reliability above required levels. This is not the case of subsea pipelines, or risers, responsible for carrying oil and gas extracted from sea bottom wells, up to the floating units on the ocean surface. These structures are very difficult to inspect and repair after launched and installed, and they are subject to significant dynamic action and cyclic loads, over long service lives. Hence, in addition to using good quality materials and welding methods, it is fundamental to inspect 100% of welds, and to remove/repair those flaws which could become critical during design life.

This gives rise to an inverse time-variant reliability problem, where parameters of the acceptance criteria must be defined in such a way that the structure or component satisfies the required reliability level along their lifetimes. Employing the previously defined acceptance criterion based on initial crack depth, the inverse problem may be written as:

$$\begin{aligned} &Find: a_{ac} \\ &which\ minimizes: -a_{ac} \\ &Subject\ to: \beta(a_{ac}, t_D, \boldsymbol{X}) \geq \beta_T\ . \end{aligned} \tag{11}$$

The minimization of $-a_{ac}$, or maximization of a_{ac}, has the objective of generating the least number of rejections or repairs, which have obvious costs. In the launching of rigid subsea risers, which are welded onboard a launching vessel, repairing a defect has significant impact in rental costs. Hence, it is of interest to maximize the acceptable flaw depth value, a_{ac}, but respecting the target reliability constraint.

In principle, solution to the optimization problem in Eq. (10) could be obtained by any numerical method for constrained optimum design, as those described in Arora [58] and other text books. A double-loop FORM approach could be employed, where the outer loop searches for the optimal value of a_{ac}, while the inner loop is responsible for computing the reliability index, for a given a_{ac}. The objective function is a very simple linear function, for which convergence can be achieved in as few as a single iteration, depending on the optimization method and on the characteristics of the constraint. The problem could also be solved by means of single-loop approaches, such as the Reliability Index Approach (RIA) or the Performance Measure Approach (PMA) [5, 59], specially developed to solve RBDO problems. However, as previously discussed, high dimensionality, highly non-Gaussian probability distributions, unstable gradients, among other issues, make difficult the application of methods such a FORM, RIA and PMA, to this kind of problem. This has driven development of the hybrid approach described in the next section.

3. PROPOSED HYBRID MCS-FORM APPROACH

The proposed approach targets determination of parameters related to acceptance criteria of initial flaws in structures. More specifically, the approach as presented herein addresses the acceptable initial crack depth so that the structure achieves a target reliability index at the end of the design life. However, it can be extended to many problems with similar characteristics, as also discussed herein. The proposed approach explores and combines advantages of Monte Carlo Simulation (MCS), with the First Order Reliability Method (FORM), with a simple root search among simulated samples. A diagram scheme of the proposed hybrid approach is presented in Figure 1.

Simulation is employed to deal with the stochastic crack propagation part of the problem: this simplifies consideration of large numbers of stochastic variables, complex non-linear crack propagation models, including load sequence effects, inspections and interventions at any time along the lifespan, and allows taking into account probability of detection curves and uncertainties in measurements. An efficient scheme for selecting the samples to be simulated is also proposed. The scheme may significantly reduce computational costs related to the solution of problems with small failure probabilities, a common scenario for real-world structural problems. The scheme has some resemblance with the Failure Sampling method proposed by Eamon and Charumas [60].

FORM is employed to handle the random variables relevant to the end of lifespan fracture problem. In this paper, only the critical crack size a_{cr} is considered as a resistance variable in the limit state function, as stated in Eq. (10). However, if one where to consider elastic-plastic fracture, critical stress intensity factor and yield stress would be relevant variables here. Considering FORM for these variables accelerates evaluation of conditional failures probabilities. In this stage, FORM may also be replaced by other reliability methods, even MCS, if the degree of limit state nonlinearity so requires. Finally, a root-finding strategy is proposed, so that the optimization problem is solved by using only the already performed simulations. This means that a single reliability analysis is sufficient to determine all the parameters of the acceptance criterion.

Overall, the approach consists of two steps (see Figure 1): 1) reliability analysis: where the failure probability is computed by using selected simulations and FORM; 2) solution of the inverse reliability problem: where the acceptance criteria parameters are determined. These steps are described in detail in the following subsections.

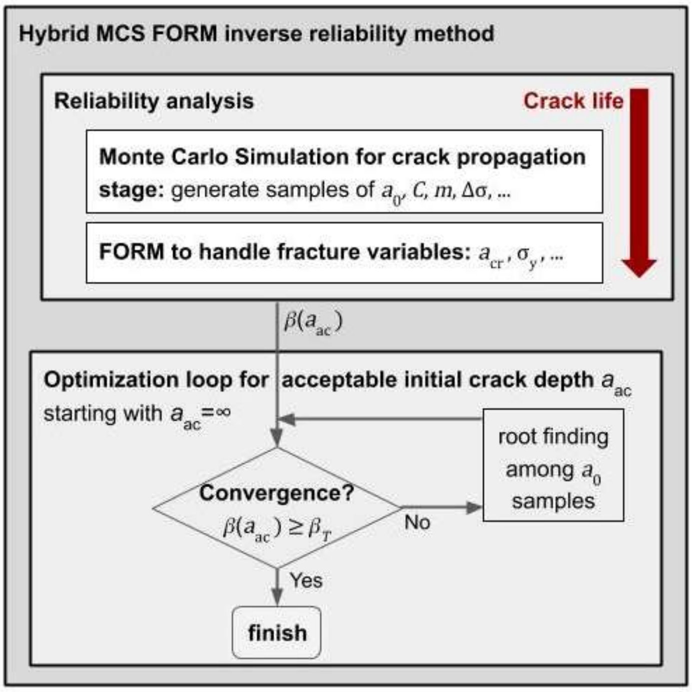

Fig. 1 – Proposed hybrid algorithm combining MCS, FORM and root finding amongst single MC sample.

3.1 Computation of the failure probability

To better understand the proposed approach, the probability of failure given in Eq. (2) is rewritten in terms of the initial crack depth, a_0:

$$P_f = \int_{-\infty}^{+\infty} P_{f|a_0} f_{a_0}(a_0)\, da, \tag{12}$$

where $P_{f|a_0}$ is the failure probability conditional on the occurrence of a given value of a_0.

Note that, for small enough values of a_0, the conditional failure probability vanishes, or is small enough to be neglected. In this case, the crack does not grow enough to significantly contribute to failure probabilities. However, as a_0 increases, the corresponding $P_{f|a_0}$ also increases, resulting in non-null, non-negligible, failure probabilities. Finally, for a_0 large enough, the conditional failure probability approaches 100%. Therefore, conditional failure probability $P_{f|a_0}$ may be classified in three distinct regions, as illustrated in Figure 2: (1) region of negligible failure probabilities; (2) region of increasing failure probabilities; (3) region of very high, close to 100%, failure probabilities.

The contribution of region (1) to the overall failure probability is negligible, while in region (3) the probability content of a_0 is usually small, because a_0 has a low probability of being in this region. Thus, solution by MCS essentially requires simulations to determine where region (2) begins and where it ends, as well as to characterize the conditional failure probability within region (2).

Let us consider again a Monte Carlo population $\boldsymbol{x}_i$, with i=1, 2, …,n_{MC}, comprised of samples randomly generated according to the joint PDF $f_{\boldsymbol{X}}(\boldsymbol{x})$ and including sampled values of a_0. For the ith simulation, the failure probability conditional on the occurrence of a_{0i} may be determined by: 1) propagating the crack from its initial depth, a_{0i}, until its final depth, a_{fi}, at the end of the lifetime; 2) computing the failure probability at the final time, conditional to a_{fi}. To simplify the notation, the failure probability conditional on the ith simulation is represented by:

$$P_{fi} = P_{f|a_{0i}} = P_{f|a_{fi}} = P\left[\left\{g\left(\boldsymbol{X}^{frac}, a_{fi}(\boldsymbol{x}_i^{prop})\right) \leq 0\right\}\right]. \tag{13}$$

In Eq. (12), the vector of sample realizations $\boldsymbol{x}_i^{prop}$ denotes those variables which are handled by MCS, in the proposed hybrid approach. Usually, these are the variables that affect the crack propagation phase, i.e., $\boldsymbol{x}_i^{prop} = \{a_{0i}, C_i, m_i, \Delta\sigma_i\}$, which is the strongly non-linear part of the problem. The probability evaluation in Eq. (13) actually refers to the end-of-life fracture problem; hence, it also refers to the failure criteria, which can be more elaborate than Eq. (10). For instance, the R6 elastic-plastic fracture criteria [46] could be incorporated here. In the proposed hybrid approach, FORM is used for computing the probability in Eq. (13), by default, due to its efficiency. The simple limit state function in Eq. (10) is linear in random variables a_{cr} and $a_n(\boldsymbol{x}_i^{prop})$; hence, the FORM solution will often be accurate enough.

Assuming the MC simulations to be statistically independent, each sample has an associated probability of occurrence of $1/n_{MC}$, and the Monte Carlo estimate for the failure probability is given by:

$$\hat{P}_f = \frac{1}{n_{MC}} \sum_{i=1}^{n_{MC}} P_{fi}. \tag{14}$$

For structural engineering applications, P_f is usually very small, and many of the P_{fi} involved in the summation are negligible. This means that not all simulations need to be performed. As long as the negligible terms P_{fi} are somehow identified, their respective simulations can be avoided. To do so, the samples of a_0 may be put in descending order, as illustrated in Figure 3, in such a way that a_{01} corresponds to the sample with the largest initial crack depth. Simulations may be performed starting from a_{01} and stopping at a_{0k} when the first conditional failure probability results null (see Figure 3).

The approach requires sorting of the samples at the initial time, in order to avoid unnecessary simulations. However, note that even if more parameters were used to describe the flaw, the samples could still be sorted. One possibility is by evaluating the limit state function, considering the flaws defined by the initial samples, and sorting the samples in ascending order, according to their respective limit state values. Samples with lower values of the limit state function at the initial time are those most probably leading to failures at the final time. Again, simulations would be performed only until the kth sample is achieved, for which the conditional failure probability results negligible. The limit state based sorting is efficient in cases where accurate enough, but not too computationally demanding, limit state functions are available. This includes many practical applications of Failure Assessment Diagrams (for example, see [46]). Also, simplified versions of the limit state function could also be used for this task, which expands the applicability of the approach.

After sorting the samples, only k simulations are necessary to compute the P_f. Thus, Eq. (14) becomes:

$$\hat{P}_f = \frac{1}{n_{MC}} \sum_{i=1}^{k} P_{fi}. \tag{15}$$

However, due to the stochastic nature of the crack propagation phenomena, given two initial cracks, the initially smaller crack could grow faster, overtaking the initially larger crack. As a result, different values of a_0 may lead to similar values of the conditional failure probability. This is illustrated by the horizontal line in Figure 4, where for a_{0i} and a_{0j}, P_{fi} may result close or equal to P_{fj}. To circumvent this, simulations must be carried out not only until the first conditional failure probability results small enough, which happens for a_{0m} in Figure 4, but until the changes on the estimated failure probability are small enough, over the last n_{stall} simulations. A tolerance on P_f change, tol_{Pf}, is considered. The more distant the points a_{0i} and a_{0j} are, which may lead to similar conditional probabilities, the higher the n_{stall} required to ensure that convergence is achieved. This convergence criterion is still directly applicable when the flaw is described by more than one parameter. After convergence, P_f may be estimated by Eq. (15).

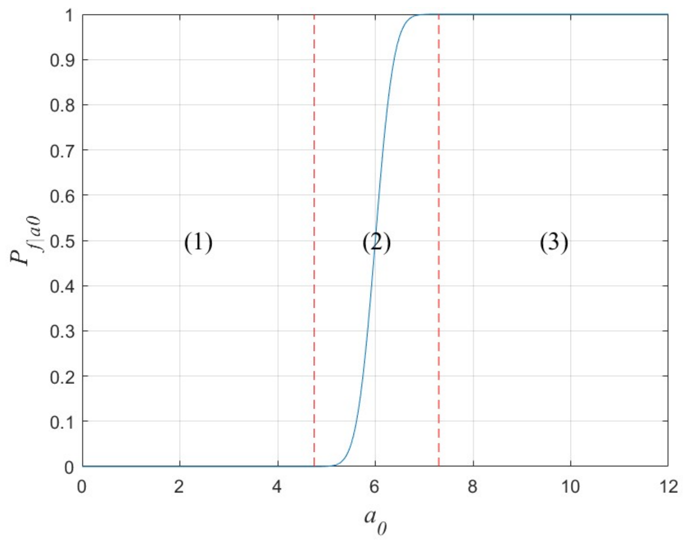

Fig. 2 – Classification of conditional failure probabilities, $P_{f|a_0}$, in terms of a_0.

.

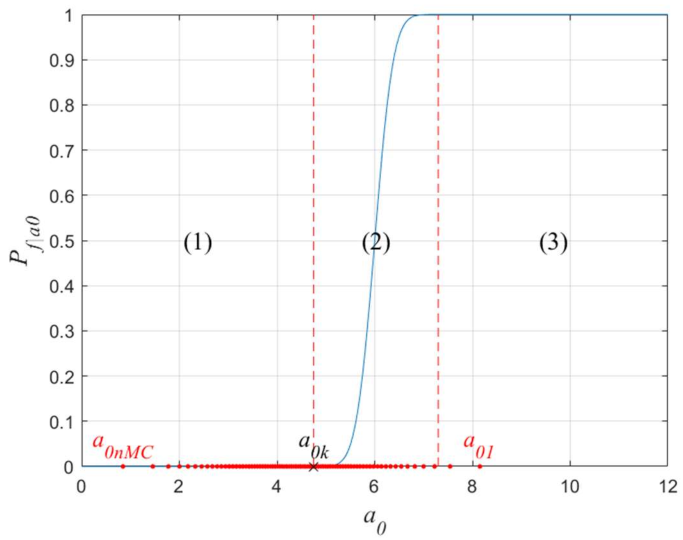

Fig. 3 – Conditional failure probabilities and samples of a_0 in descending order.

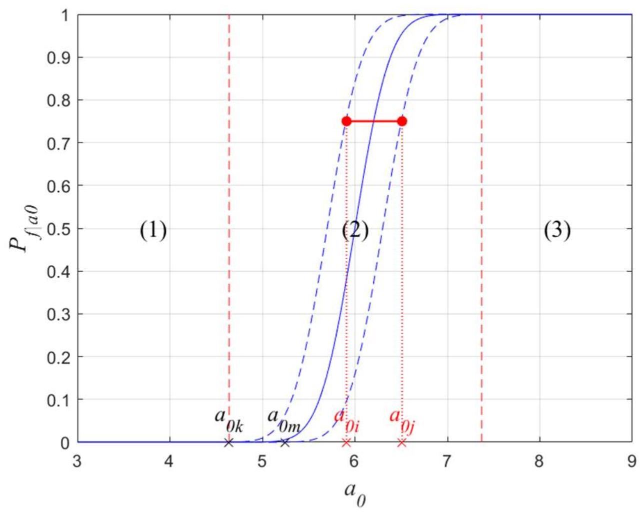

Fig. 4 – Similar conditional failure probabilities for different values of a_0.

3.2 Determination of acceptance criterion parameters

The second and last step of the proposed approach relates to finding the parameters of the acceptance criterion, so that at the end of the design life, the structure presents a reliability index, β, equal to or larger than the target, β_T, or, in terms of failure probabilities, $P_f \leq P_{fT}$. For the simple problem taken as an example, this corresponds to finding the largest value of a_{ac} for which the structure or component satisfies the reliability constraint.

Consider that a sufficient number of MCS samples are drawn, and that $\hat{P}_f$ is calculated following Eq. (15). If $\hat{P}_f \leq P_{fT}$, no acceptance criterion would be necessary: safety of the structure already complies with requirements, any inspections or interventions would lead to unnecessary costs. On the other hand, if $\hat{P}_f > P_{fT}$, the acceptance criterion must be determined and applied.

One efficient way to determine parameters of the acceptance criterion is by using the previously performed simulations. The rejection of an actual flaw, which is greater than a_{ac}, is equivalent to rejecting the corresponding MC sample, which also changes the evaluated failure probability. Smaller values of a_{ac} lead to greater number of rejected samples and larger fabrication costs, but also leads to smaller failure probabilities.

As previously stated, each sample has a probability of occurrence, P_o, of $1/n_{MC}$. If a sample is rejected (as the actual flaw would), its probability is redistributed to the other MCS samples. Hence, after one rejection, the probability of occurrence of the remaining samples given the rejection, $P_{o|R}$, becomes:

$$P_{o|R} = \frac{1}{n_{MC}} + \frac{\frac{1}{n_{MC}}}{(n_{MC}-1)} = \frac{1}{(n_{MC}-1)}. \tag{16}$$

In other words, the acceptance/rejection criterion has the effect of truncating the initial joint density distribution, which is equivalent to eliminating some samples of the Monte Carlo population. The Monte Carlo estimate of the failure probability after truncation is computed by using the remaining samples and their corresponding conditional failure probabilities.

Note that repairs could also be considered in the proposed approach. In this case, instead of eliminating the sample, the rejected sample would be submitted to a repair procedure and give rise to a modified simulation with updated conditional probability, or to a number of simulations in case a probabilistic repair takes place.

For the example based on only a_0 and the respective a_{ac}, and disregarding repairs, the simulations give a discrete set of failure probability values P_{fi} over a_0, with the samples sorted in descending order. Thus, for a_{ac} within the interval $[a_{0j}\ a_{0j+1}[$, the acceptance criterion has the effect of eliminating all simulations for which $i \leq j$. The failure probability after truncation, P_{fTR}, is given as a function of j by:

$$P_{fTR}(j) = \frac{1}{(N_{samp}-j)} \sum_{i=j+1}^{k} P_{fi}. \tag{17}$$

Solution of the inverse reliability problem is obtained by imposing $P_{fTR}(j) \leq P_{fT}$, that is, the failure probability after application of the acceptance criterion is smaller or equal to the target. However, the discrete description of the P_f provided by MCS only allows determining an interval for a_{ac}.

This problem may be seen as a root-finding problem, which aims at finding the roots of the error given by:

$$error(j) = P_{fTR}(j) - P_{fT}. \tag{18}$$

Among the many root-finding methods available in the literature [61], the bisection method is chosen herein to solve this problem. Assuming that the root is in the interval $[a_{0p}, a_{0q}]$, for which $P_{fTR}(p) \geq P_{fT}$ and $P_{fTR}(q) \leq P_{fT}$, the bisection method works by narrowing the interval until the solution is found with a specified tolerance. In the original version of the bisection method, the interval is updated by replacing either a_{0p} or a_{0q} by $(a_{0p} + a_{0q})/2$. The version used herein converges a little faster by looking to the indexes of the samples, instead of their actual values. In each iteration, either p or q is updated by $floor((p + q)/2)$, keeping $P_{fTR}(p) \geq P_{fT}$ and $P_{fTR}(q) \leq P_{fT}$, where the function *floor*(.) rounds its argument towards the closest smaller integer. In the problem at hand, the search stops when the interval $[j, j+1]$ is found, for which $error(j) \geq 0$ and $error(j+1) < 0$. The respective tolerance on the solution is directly related to the spacing between the samples. Figure 5 illustrates convergence of the method over six iterations.

Finally, a_{ac} is obtained by linear interpolation in the last interval, considering the respective values of the truncated failure probability $[P_{fTR}(j), P_{fTR}(j+1)]$ and the target failure probability.

The initial interval can be taken as $[a_{00}, a_{0k}]$, with a_{00} slightly above a_{01}. Remembering that a_{01} corresponds to the sample with the largest initial crack depth, if $a_{ac}=a_{00}$ no rejections occur and the failure probability remains the one given by Eq. (15). On the other hand, for $a_{ac} = a_{0k}$, all simulated samples are rejected and the failure probability vanishes. For this initial interval, $P_{fTR}(p) = P_{fTR}(0) = \hat{P}_f$ and $P_{fTR}(q) = P_{fTR}(k) = 0$, so that the target failure probability is within the initial interval, *i.e.* $P_{fT} \in [0, \hat{P}_f]$.

When the acceptance criterion is based on more parameters, optimization algorithms may be necessary to determine the solution. Nevertheless, the procedure still uses the previous simulations and may be performed in an efficient way.

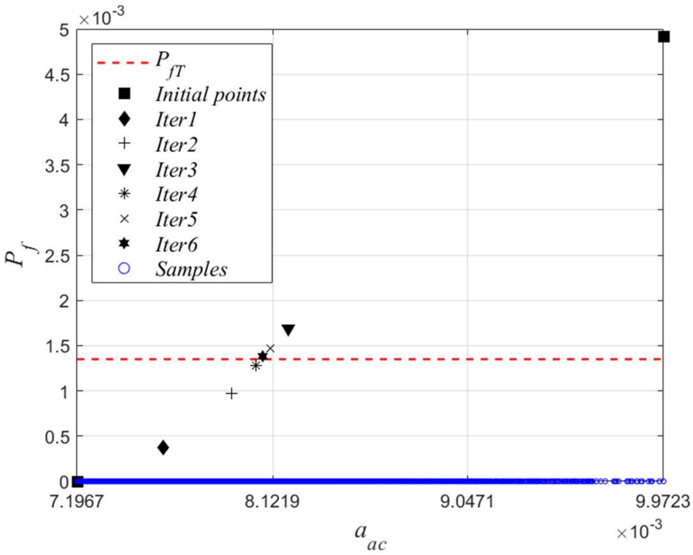

Fig.5 – Illustration of the root-finding scheme via Bisection.

4. NUMERICAL RESULTS

Due to space limitations, numerical results will be presented at the conference and in the extended journal submission.

5. CONCLUDING REMARKS

Classical approaches to inverse reliability and reliability-based design optimization problems require evaluation of limit state gradients. This can be a problem when dealing with non-linear crack propagation problems involving millions of load cycles, hence millions of random variables. In this paper, a hybrid approach was proposed for solving this type of problems. The proposed hybrid approach explores and combines the best of Monte Carlo Simulation (MCS), FORM and the bisection method. The non-linear crack propagation phase of the problem is handled by MCS; the final fracture problem is handled by FORM; and the allowable crack size, required in order to impose a minimum lifetime reliability, is obtained by root-finding, among the set of initial crack size samples. Efficiency is achieved by classifying initial crack size samples, and computing only those terms effectively contributing non-zero probabilities to the integral. Efficiency is also achieved by solving the optimization problem using the information acquired from a single Monte Carlo run. As shown in the manuscript, efficiency of the proposed Hybrid approach improves when target reliabilities are large, as expected in practical structural engineering applications.

REFERENCES:

[1] Der Kiureghian A, Zhang Y, Li CC. Inverse Reliability Problem. Journal of Engineering Mechanics 1994; 120(5): 1154-1159. https://doi.org/10.1061/(ASCE)0733-9399(1994)120:5(1154)

[2] Li H, Foschi RO. An inverse reliability method and its application. Structural Safety 1998; 20: 257-270. https://doi.org/10.1016/S0167-4730(98)00010-1

[3] Mínguez R, Castillo E, Hadi AS. Solving the inverse reliability problem using decomposition techniques. Structural Safety 2005; 27: 1–23. https://doi.org/10.1016/j.strusafe.2004.04.001

[4] Frangopol DM. Multicriteria reliability-based structural optimization. Structural Safety 1985; 3(1): 23–8. https://doi.org/10.1016/0167-4730(85)90004-9

[5] Aoues Y, Chateauneuf A. Benchmark study of numerical methods for reliability-based design optimization. Structural and Multidisciplinary Optimization 2010; 41: 277–94. https://doi.org/10.1007/s00158-009-0412-2

[6] Beck AT, Gomes WJS. A comparison of deterministic, reliability-based and risk-based structural optimization under uncertainty. Probabilistic Engineering Mechanics 2012; 28: 18–29. https://doi.org/10.1016/j.probengmech.2011.08.007

[7] Beck AT, Gomes WJS, Lopez RH, Miguel LFF, 2015: A comparison between robust and risk-based optimization under uncertainty, Struct. Multidisc. Optim. 52, 479-492, 2015. https://doi.org/10.1007/s00158-015-1253-9

[8] Torii AJ, Lopez RH, Miguel LFF. A second order SAP algorithm for risk and reliability based design optimization. Reliability Engineering & System Safety 2019; 190: 106499. https://doi.org/10.1016/j.ress.2019.106499

[9] Moustapha M, Sudret B. Surrogate-assisted reliability-based design optimization: a survey and a unified modular framework. Structural and Multidisciplinary Optimization 2019; 60: 2157–2176. https://doi.org/10.1007/s00158-019-02290-y

[10] Zhang Y, Der Kiureghian A. Two Improved Algorithms for Reliability Analysis. In: Rackwitz R, Augusti G, Borri A, editors. Reliability and Optimization of Structural Systems. IFIP - The International Federation for Information Processing. Boston: Springer; 1995, p. 297-304. https://doi.org/10.1007/978-0-387-34866-7_32

[11] Winterstein S, Ude TC, Cornell CA, Bjerager P, Haver S. Environmental parameters for extreme response inverse FORM with omission sensitivity. In: Proceedings of International Conference on Structural Safety and Reliability 1993, Innsbruck, ICOSSAR-93.

[12] Maes MA. Ignorance Factors Using Model Expansion. J Eng Mech 1996; 122(1):39-45. https://doi.org/10.1061/(ASCE)0733-9399(1996)122:1(39)

[13] Sadovsky Z. Discussion on: An inverse reliability method and its application. Structural Safety 2000; 22: 97-102. https://doi.org/10.1016/S0167-4730(99)00039-9

[38] Ditlevsen O, Madsen HO. Structural Reliability Methods. Technical University of Denmark; 2007.

[39] Melchers RE, Beck AT. Structural reliability analysis and prediction. 3rd ed. New York: John Wiley and Sons; 2018. https://doi.org/10.1002/9781119266105

[40] Engelund S, Rackwitz R. A benchmark study on importance sampling techniques in structural reliability. Structural Safety 1993; 12: 255–76. https://doi.org/10.1016/0167-4730(93)90056-7

[41] Maes MA, Breitung K, Dupuis DJ. Asymptotic importance sampling. Structural Safety 1993; 12: 167-186. https://doi.org/10.1016/0167-4730(93)90001-H

[42] Au S-K, Beck JL. Estimation of small failure probabilities in high dimensions by subset simulation. Probabilistic Engineering Mechanics 2001; 16(4): 263–277. https://doi.org/10.1016/S0266-8920(01)00019-4

[43] Beck AT, Silva Jr. CRA. Strategies for Finding the Design Point under Bounded Random Variables. Structural Safety 2016, 58, 79-93. https://doi.org/10.1016/j.strusafe.2015.08.006.

[44] Carpinteri A. Handbook of Fatigue Crack: Propagation in Metallic Structures. Amsterdam: Elsevier; 1994. https://doi.org/10.1016/C2009-0-13210-5

[45] Schijve, J. Fatigue of Structures and Materials. 2nd ed. Netherlands: Springer; 2009. https://doi.org/10.1007/978-1-4020-6808-9

[46] Anderson TL. Fracture Mechanics: Fundamentals and Applications. 3rd ed. Boca Raton: Taylor & Francis Group; 2005. https://doi.org/10.1201/9781420058215

[47] Beck AT, Melchers RE. Overload Failure of Structural Components under Random Crack Propagation and Loading - a Random Process Approach -, Structural Safety 2004, 26, 471-488.

[48] Berens A, Hovey P. Evaluation of NDE reliability characterization. Report no AFWAL-TR-81-4160, vol. 1. University of Dayton Research Institute; 1981.

[49] Guedes Soares C. Quantification of model uncertainty in structural reliability. In: Guedes Soares C., editor. Probabilistic methods for structural design. Dordrecht: Springer; 1997, p. 17-37. https://doi.org/10.1007/978-94-011-5614-1_2

[50] Madsen HO. Stochastic Modeling of Fatigue Crack Growth and Inspection. In: Guedes Soares C., editor. Probabilistic methods for structural design. Dordrecht: Springer; 1997, p. 59-83. https://doi.org/10.1007/978-94-011-5614-1_4

[51] Gomes WJS, Beck AT. Optimal inspection planning and repair under random crack propagation. Engineering Structures 2014; 69: 285–296. http://dx.doi.org/10.1016/j.engstruct.2014.03.021

[52] Sobczyk K, Spencer BF. Random fatigue: from data to theory. London: Academic Press; 1992.

[53] Paris PC, Erdogan F. A critical analysis of crack propagation laws. J Basic Eng 1963; 85(4): 528–34. https://doi.org/10.1115/1.3656900

[54] Newman Jr JC, Raju IS. An Empirical Stress-Intensity Factor Equation for the Surface Crack. Engineeng Fracture Mechanics 1981; 15(1-2): 185-192. http://dx.doi.org/10.1016/0013-7944(81)90116-8

[55] Mechab B, Serier B, Bachir Bouiadjra B, Kaddouri K, Feaugas X. Linear and non-linear analyses for semi-elliptical surface cracks in pipes under bending. International Journal of Pressure Vessels and Piping 2011; 88(1): 57:63. https://doi.org/10.1016/j.ijpvp.2010.11.001

[56] Predan J, Mocilnik V, Gubeljak N. Stress-intensity factors for circumferential semi-elliptical surface cracks in a hollow cylinder subjected to pure torsion. Engineering Fracture Mechanics 2013; 105: 152–168. http://dx.doi.org/10.1016/j.engfracmech.2013.03.033

[57] Raju IS, Newman Jr JC. Stress-intensity factor for circumferential surface cracks in pipes and rods under tension and bending loads. NASA Technical Memorandum 87594, 1985.

[58] Arora JS. Introduction to Optimum Design. 3rd ed. Oxford: Elsevier, 2012.

[59] Lopez RH and Beck AT. Reliability-based design optimization strategies based on FORM: a review. J. Braz. Soc. Mech. Sci. & Eng. [online] 2012; 34(4): 506-514. http://dx.doi.org/10.1590/S1678-58782012000400012.

[60] Eamon C, Charumas B. Reliability Estimation Of Complex Numerical Problems Using Modified Conditional Expectation Method. Computers and Structures 2011; 446(89): 181-188. https://doi.org/10.1016/j.compstruc.2010.09.002