Estimation of reliability prior distribution based on generic data and experts' opinion: a case study in the O&G industry

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ABSTRACT

In the Bayesian framework, the prior knowledge about a reliability measure can be updated as new information is obtained. Theoretically, the possibly more spread initial state of knowledge about the system reliability may gradually reduce as new field and/or test data may narrow the prior estimate's uncertainty. However, these data are rather scarce and expensive to obtain, especially for technologies under development in industries such as O&G. In this context, the intrinsically limited prior knowledge on the system reliability is only available in terms of generic databases and expert opinions. In those cases, the prior distribution has a major strength in the reliability estimate obtained at the end of the development of new technology. Bayesian reliability analysis strongly relies on estimating appropriate informative prior distributions. Thus, we propose an approach that does not require direct elicitation of parameters to define informative prior distributions using expert's opinions and/or generic data at the system level of new equipment. Specifically, the method of moments and maximum-entropy are adopted to aggregate information at the system level from generic sources and expert opinions for the estimation of the prior probability distributions of the reliability parameters. Finally, we present a case study of specific completion equipment to be installed in a Brazilian oil field considering an O&G generic database and expert opinion.

1. INTRODUCTION

Oil & Gas (O&G) wells operation usually involves extremely complex equipment, in which reliability estimation is paramount to ensure sustained production and reduce maintenance costs. However, reliability data is frequently absent, scarce, or insufficient because tests experiments are usually very costly. In this context, Bayesian inference allows aggregating and updating prior knowledge as new information is acquired [1].

Thus, prior distributions are a key component for Bayesian analysis. Non-informative prior affects the likelihood information as low as possible. However, when limited data is available, the likelihood constructed is sometimes weak and, using non-informative prior may end in a biased posterior distribution with high uncertainty. Therefore, it is essential to incorporate as much information as possible to build the prior distribution in order to improve the reliability estimation accuracy [2].

To define the prior distribution for the reliability of equipment under development, we propose a methodology that does not require direct elicitation of probability distribution parameters but rather uses expert opinion and/or generic data for the Fault Tree (FT)'s top event (the system level). Thus, the proposed methodology allows keeping elicitation simple and intuitive. To that end, we adopted two approaches, one of them is based on the method-of-moments (MM) [3] and the other relies on the maximum-entropy (ME) method [4].

The remainder of this paper unfolds as follows. Section 2 presents the proposed methodology, describing the elicitation methods and MM and ME approaches suitable for distinct types of events. Section 3 presents an O&G case study applied to a novel completion expansion packer introduced in well completion. Finally, Section 4 provides concluding remarks.

2. DESCRIPTION

Our proposed methodology is illustrated in Figure 1, in which the challenge is to propagate 'downward' from a top event (E_T) the information gathered from generic database throughout the failure modes (E_{FM_1} and E_{FM_2}) until the basic events (E_A, E_B, E_C, E_D and E_E) of the novel technology. To that end, we consider expert opinions and two distinct approaches to define the prior distribution for each basic event. Finally, these distributions are used in a Monte Carlo simulation algorithm to propagate 'upward' the

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uncertainty from the basic events and obtain an uncertainty distribution of the system's reliability. Then, the results may be compared to the desired target reliability measure to assess the risk associated with the equipment's application.

Since in FT the events are related to different consequences and probabilities, the event i contributes with a weight w_i to the immediate upper event. These weights allow us to build relations between the top and basic events used to estimate the equipment reliability. In this work, only the weights for each event are elicited, avoiding the direct estimation of parameters. In addition, as distinct experts may have different knowledge about the new equipment, one may assign specific relevance factors for each expert (i.e., the j-th specialist may receive a relevance factor, r_j, validated by other reliable sources such as senior experts or a consensus). Then, the quantitative responses of experts with $r_j = x$, $\forall x \in \mathbb{Z}_+^*$, are considered x times in order to compute a weighted median value.

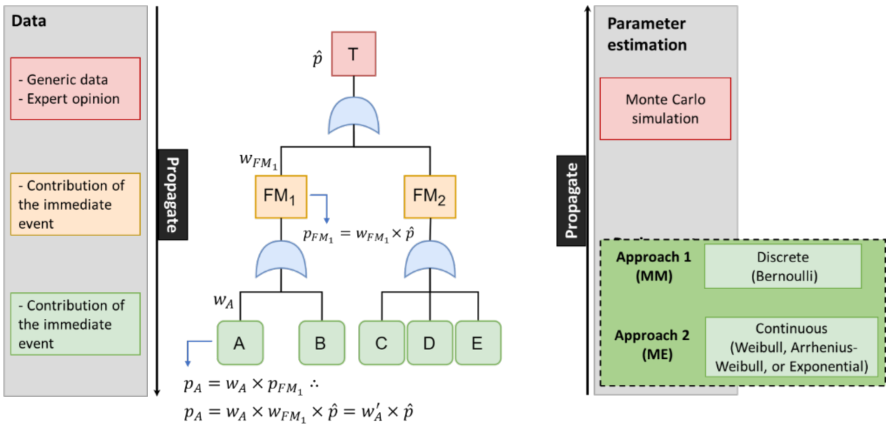

Fig 1 - Overview of the proposed methodology and how the contributions of the basic events are computed.

The elicited contributions define the relations between the system level and the other events. For example, in Figure 1, given $\hat{p}$ (i.e., the estimate of the failure probability related to E_T), one defines the probability of the failure mode E_{FM_1} as $p_{FM_1} = \hat{p} \times w_{FM_1}$ (I), where w_{FM_1} represents the contribution of E_{FM_1} to $\hat{p}$. Analogously, the failure probability of basic event E_A can be defined as $p_A = p_{FM_1} \times w_A$ (II), where w_A is the weight of E_A to p_{FM_1}. Then, combining (I) and (II), $p_A = \hat{p} \times w_{FM_1} \times w_A$, and setting $w'_A = w_{FM_1} \times w_A$ as the weight of E_A to $\hat{p}$, we get $p_A = \hat{p} \times w'_A$.

The system's reliability can be represented by a multilevel reliability model (MRM). Here, the MRM model uses the logic gates of the FT, in which each basic event i (E_i) has a set of parameters θ_i that model the probability of occurrence of the corresponding mechanism/cause of failure. Then, the reliability function of the entire system is a function of θ_i, which translate the possible pathways leading to equipment failure for example over a mission of T years [5].

We adopt distinct approaches depending on the characteristics of the basic event: Approach 1 is adopted if success/fail event are considered, and Approach 2 is adopted if continuous distributions describe the basic events. In Approach 1, the occurrence of an event E_i is assumed to be well described by a Bernoulli distribution with parameter p_i. We assumed there is uncertainty in p_i, modeled by $\pi(p_i)$ as a beta distribution $B(\alpha_{p_i}, \beta_{p_i})$, $\forall i$. In this approach, in order to obtain, the mean, μ_{p_i}, and variance, ${\sigma_{p_i}}^2$, the PERT distribution is adopted, where a_{p_i} is the optimistic estimate, m_{p_i} is the most likely estimate, and b_{p_i} is the pessimistic estimate of p_i [6]. Thus, from the three estimates, the expected value and variance of p_i is calculated (Equations 1 and 2, respectively). These values allow us to analytically estimate the prior distribution through MM. The estimate $\hat{p}$ is the most likely value and we assume that percentile 1, P_1, and percentile 99, P_{99}, are the optimistic and pessimistic values, respectively. As mentioned, basic event E_i contributes with a weight w_i to upper event.

Thus, the relations $m_{p_i} = \hat{p} \times w_i'$, $a_{p_i} = P_1 \times w_i'$, and $b_{p_i} = P_{99} \times w_i'$ were used to compute μ_{p_i} and σ_{p_i}, where w_i' is the contribution of event i to the top event.

$$\mu_{p_i} = \frac{a_{p_i} + 4m_{p_i} + b_{p_i}}{6} \tag{1}$$

$$\sigma_{p_i}{}^2 = \left(\frac{b_{p_i} - m_{p_i}}{6}\right)^2 \tag{2}$$

In Approach 2, the occurrence of event E_i is described by a continuous distribution, and ME method is adopted. ME method involves maximizing the entropy measure H (Equation 3), where $\pi(\theta_i)$ is the probability density function (PDF) for the parameters of event i and Θ_i is the parameter space of θ_i.

$$\text{maximize } H = \int_{\Theta_\mathrm{i}} -\pi(\theta_i) \times \log[\pi(\theta_i)]\, d\theta_i \tag{3}$$

As constraints we consider that the expected value as well as the percentiles 5 and 95 of each event i must be equal to the expected value, percentiles 5 and 95 of the reliability R_T of the top event E_T weighted (exponentially) by its specific contribution w_i'. Thus, we use a Particle Swarm Optimization (PSO) algorithm to obtain the solution of the maximization problem.

3. DISCUSSION

The proposed methodology is applied to a case study of a novel expansible production packer, which is a common completion equipment of the O&G industry. The FT related to the equipment failure during its installation, contains twelve basic events. The MRM is obtained by the multiplication of the reliability models of the basic events since all logic gates of the FT are of the "OR" type. In this case study, the only initially available information is for the system level accessible in generic database by Wellmaster report: the number of production packers installed and the number of failures during its installation are 5,730 and 16 respectively.

The FT diagram related to the equipment failure during its operation contains seven basic events. The MRM model was defined by multiplying different reliability models: exponential model, with parameter λ_i, Weibull model, with parameters α_i and β_i, and Arrhenius-Weibull model with parameters a_i, b_i, and β_i. As in the installation case, the only available information is the data presented by the Wellmaster. The report provides the scale (α) and shape (β) parameters of the Weibull for the time to failure related to the failure modes annulus communication, 63,521.0 and 0.605, and column-annulus communication, 860.2 and 1.372, during the operation. summarizes this information.

After eliciting and computing the contributions of the basic events we determined the prior distributions for each basic event. Then, we used a Monte Carlo simulation algorithm to perform 'upward' propagation, i.e., to propagate the uncertainty through the FT until the top event, determining the probability of failure in installation and during operation (considering a mission time of 27 years) for the novel O&G packer. Thus, for the real case study, equipment reliability during its installation, $R(0)$, is represented in Fig 2a and the reliability estimation, R(27), is shown in Fig 2b. For the installation the results meet the metric recommended by API 17N [7] as the failure probability tends to 0. For the operation, the mean value for the reliability is 93.48%, while the probability that the reliability is less than 90% (red line) is 28.1%.

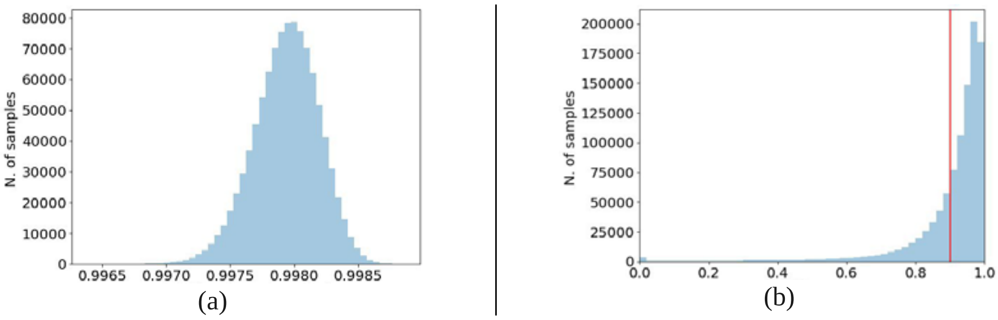

Fig 2 - Estimate of the equipment reliability for 27 years of operation, $R(27)$.

4. CONCLUSION

The proposed approaches do not involve directly eliciting the expert opinion about the hyperparameters; this allows the adoption of different distributions to describe the basic events without hindering understanding during the elicitation process. Also, we adopted an elicitation procedure to fit the Wellmaster results in the specific scenarios for a novel expandable production packer of the O&G industry. The elicitation process engaged specialists from different areas involved in the equipment development process, allowing for a balance between pessimistic and optimistic analyses. Moreover, the procedure was based on the analysis of component failure mechanisms. Although the system is new, the components and materials are "old acquaintances" of the experts. The methodology considers distinct FTs to deal with different stages of equipment life cycle and, in the case study, the two analyzed stages were installation and operation. The results obtained for the equipment installation and operation can be used to estimate the equipment reliability, evaluating the performance using standards such as API 17N 2018. After the presented analysis, the level of uncertainty for $R \geq 90\%$ is 28.82% and, thus, above the maximum limit (20%). This is somehow expected because we are only considering the prior distribution of the equipment based on generic data and expert opinion.

5. ACKOWLEDGEMENTS

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6. REFERENCES

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