OPTIMAL INSPECTION INTERVAL BASED ON CONSTANT FAILURE RATE

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ABSTRACT

The maintenance task interval is an important step for maintenance plan effectiveness. Assuming that maintenance task exist to prevent or predict failures, when the task frequency is not properly determined, it may be excessively, reducing equipment availability and increasing maintenance costs, or it may not be effective by not capturing failure before it happens, therefore, reducing equipment availability and increase maintenance costs. In this way, maintenance task interval must be properly defined.

This paper proposes an intermediate mathematical model to assist maintenance and reliability engineer to establish the best inspection interval when there is just available mean time to failure (*MTTF*), mean time between failures (*MTBF*) or constant failure rate (λ) parameters. This model uses the concept of test interval for failure finding enhanced by costs parameters used on optimal preventive replacement interval and optimal inspection interval to establish the optimal inspection interval that minimize equipment failure costs. In addition to this feature, this model can be used to calculate the risk taken on the maintenance task deferment.

1. INTRODUCTION

In Oil and Gas industry, most of maintenance task intervals are estimate based on the expert's judgment or manufacturer recommendation. It is a common sense that the manufactures are quite too conservative when defining the maintenance tasks periodicity, since they cannot control the equipment operation context. In other words, the manufacture recommendation should cover a variety of operation condition.

Looking into the expert's judgments, it is important to highlight there is a lot in variables that could influence their judgment. Some are quite more conservative, other are more risk taken. Some has more experience about the failures modes and the equipment failures frequency, others not much. Some are influenced by the company´s culture or other opinions, while some are very loyal to their knowledge and believes. In other words, the maintenance task periodicity tends to be biased by the company's team that have defined the maintenance tasks periodicity.

Many different approaches can be used to obtain maintenance task intervals. The quantitative approach methods, which take into consideration the equipment or component reliability curves, must take into consideration objective functions for costs and availability. Mathematical models, like optimal preventive replacement interval and optimal inspection interval, are based on cost or availability balancing functions to failures and preventive replacement. These models are conceived to be used only with increasing failure rate functions. On the other hand, mathematical models to determine intervals for failure finding tasks, may be used with a constant failure rate, but do not take into consideration costs parameters, which is wise since they are committed with safety.

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2. OPTIMUM MAINTENANCE INTERVALS OVERVIEW

2.1 Introduction

Calculation of optimum maintenance intervals is one of the biggest challenges in a maintenance program. The use of incorrect intervals can lead the equipment to operate in non-profit scenario even though maintenance strategy was adjust correctly. Many qualitative and quantitative methods exists in literature to determine the optimum intervals, since the most easy method like Age Exploration – applied only when good statistical data is not available and experience to guess at task intervals is really the only option – until advanced statistical methods which uses distributions functions obtained from equipment life data (SILVA, 2017, p. 63).

2.2 Age Exploration

The age exploration (AE) technique is strictly empirical. It works like this illustrative example: supposing an initial overhaul interval for a fan motor is 3 years and after the first overhaul, inspections reveal no such wear out or aging signs, than the initial interval is increased automatically by 10%. Repeat the process, continuing until, on one of the overhauls, incipient signs of wear out or aging are inspected. At this point, the AE process stops, perhaps back off by 10%, and define this as the final task interval (SMITH; HINCHCLIFFE, 2003, p. 126–127).

2.3 Cost-Based Models

If cost, due to the high cost of replacement after failure, is the appropriate criterion to be applied as objective function to determine preventive maintenance interval, then a cost-based model is desired. A model proposed by (BARLOW; HUNTER, 1960) determine the optimum age time for preventive maintenance action which minimizes global costs:

$$t_c^* = \underset{t}{min}\, f_c(t) = \frac{C_p R(t) + C_u F(t)}{\int_0^t R(t)dt} \tag{1}$$

where:

t^*_c: optimum age cost-based preventive maintenance interval;
$f_c(t)$: age preventive maintenance cost function;
$R(t)$: reliability function;
$F(t)$: probability of failure function;
C_p: cost of preventive task;
C_u: cost of corrective task;
t: time.

2.4 Constant Failure Rate Models

The constant failure rate models methodology applied to determine optimal maintenance intervals was developed to work around on a commonly companies' weakness of data collection. It is very usual to see companies collecting failure at a time interval to calculate *MTTF* or not collecting any kind of data at all. For those that do not collect data, [OREDA, 2015] reference could be a great source of data. The OREDA or *MTTF* can just provide the failure rate parameter λ, which represents a constant failure rate.

It is known that some of those failures rates presented by OREDA or when calculated by *MTTF* approach, could be better represented by an increase failure rate function if proper collected and analyzed. In absence of better data, the methodology described herein aim to move from the qualitative approach represented by the expert judgment and manufacturer recommendation to a quantitative approach based on cost balancing when the company just have the *MTTF* information on hand.

To be able to apply this methodology is imperative to assume the following assumptions:

- the probability to be in a failed state at time *t* is equal to the probability of having failed before *t*, i.e., randomness. Therefore, the failure rate is constant with time, equipment does not age;
- inspection is perfect, it always detects an existing failure;
- inspection and repair are perfects, and the duration is negligible or too short compared to the interval between inspection;
- after inspection, the equipment is assumed to be as good as new;
- failure rate represents the equipment failure behavior for the specific maintenance inspection.

It is important to clarify that this model are not suitable for safety systems due to cost minimizations objectives purposes.

3. OPTIMAL INSPECTION MODEL FOR CONSTANT FAILURE RATE

3.1 Mean Probability of Failure Function

The exponential cumulative distribution function,

$$F(t) = 1 - e^{-\lambda t} \tag{2}$$

is applied to modeling failures with randomness, i.e., the modeled equipment does not age. In terms of inspection, it means that the chance of an item fail after the inspection is equal to any time before the inspection.

The conditional reliability function,

$$R(T,t) = \frac{R(T+t)}{R(T)} = \frac{e^{-\lambda(T+t)}}{e^{-\lambda T}} = \frac{e^{-\lambda T}e^{-\lambda t}}{e^{-\lambda T}} = e^{-\lambda t} \xrightarrow{yieds} F(t) = 1 - R(t) \xrightarrow{yields} F(t) = 1 - e^{-\lambda T} \tag{3}$$

calculates the probability that an item will survive in the next desired time assuming it has survived for a specific time. Observing the results in (3): the exponential reliability conditional function is equal to the exponential reliability function. In other words, the probability that an item will fail, in the next desired time assuming it has survived for a specific time, is equal to the cumulative probability of failure at the entire evaluated period.

The exponential failure rate function,

$$\lambda(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} \xrightarrow{yields} \lambda \tag{4}$$

yields to a constant failure rate parameter. It means the failure rate is the same to the entire period of interest.

Due to the behavior of the exponential distribution, one can conclude that this function models item that not age. Thus, for any time in the period of interest, the failure rate is the same. In other words, the probability that an item will failure in the next time is the same the time before. Based on this definition, the equipment (inspected items) need to be consider as good as new after inspection since the test or inspection is assume to be perfect (it always detects an existing failure). Therefore, if failure is detected, the repair is done and considered perfect as well. Its duration is negligible or too short compared to the interval between test or inspection.

It is important to highlight that inspection or test performed during the maintenance task is assumed be capable to identify any degraded or incipient failure modes that could lead to the equipment failure. Note that failure rate λ used in the model needs, as close as possible, represents the failure mode. As stated before, the probability that an equipment fails is equal at any time within the inspection interval. Based on that, and in order to avoid be too conservative, this methodology assumes the inspection time is the mean of the exponential cumulative distribution function. This assumption is also in accordance with the Det Norske Veritas (DNV) methodology based on [IEC 61508, 2010] for periodic test of protection system and according to [MOUBRAY, 2002, p. 177].

Applying the mean value theorem for definite integrals to the exponential probability function (2) yields to:

$$\bar{F}(T) = 1 - \frac{1}{\lambda t}\left(1 - e^{-\lambda t}\right) \tag{5}$$

this function can be used to define the equipment inspection interval based on *MTTF*.

3.2 Optimal Inspection Interval for Minimal Cost

Minimize the long-term maintenance cost is pursued by the majorities of the maintenance manager. Due to that, eq. (5) is combined with the cost of the failure and the cost of the inspection to minimize total inspection routine cost. The total inspection routine cost, *TIRC*:

$$TIRC = I_{RC} + E_{FC} \tag{6}$$

where:

TIRC: total inspection routine cost;
I_{RC}: inspection routine cost;
E_{FC}: expected failure cost.

is the sum of inspection routine cost and the expected failure cost. The inspection routine cost:

$$I_{RC} = \frac{E_{pu}}{I_{pu}} \cdot I_c \tag{7}$$

where:

E_{pu}: evaluation period;
I_{pu}: inspection periodicity;
I_C: inspection cost.

must be defined for a specific period since it is a periodic activity and presents the total inspection cost for evaluated period. The evaluation period unit can be define, for example, as the time where the exponential cumulative probability of failure is equal to 95%. This is obtained by applying the inverse of the exponential cumulative distribution function at 95% of probability of failure.

The expected failure cost:

$$E_{FC} = \bar{F}(t_i)\,\mathrm{U_{cc}} \tag{8}$$

where:

$\bar{F}(t_i)$: mean of cumulative probability function for exponential distribution;
U_{CC}: failure cost;
t_i: time for ith interval.

is calculated by the mean cumulative probability function (5) of the failure multiplied by the cost of the failure. Considering the equipment "as good as new" after inspection, the repair is assumed perfect. Therefore, the probability of failure in each inspection interval is the same.

The cost variable set up is very important to ensure accurate results. Considering the cost composition, one will inform to the model, the weight of inspection task and unplanned corrective task. The inspection cost composition encompasses the manhours cost, spare parts costs, rental tools cost, production losses whether the inspection task disturbs the facility operation.

If the inspection causes production unavailability, this production loss cost should be considered in the inspection cost composition. However, if the inspection could be proper planned to be performed in an operational window, thus, it does not cause production loss.

The cost of unplanned corrective task encompasses the corrective maintenance cost itself to restore the equipment function and the failure effect cost. The corrective maintenance cost encompasses the manhours cost, spare parts cost, third part cost, rental tools cost in emergency context.

4. RESULTS

The example used to apply the mathematical models to obtain the optimal inspection interval with minimal inspection routine cost, uses the following data parameters:

$$\begin{gathered} MTTF = 20\ months \\ \lambda = 0.05 \\ E_{pu} = 60\ months \\ I_C = \$1{,}000.00 \\ U_{CC} = \$50{,}000.00 \end{gathered} \tag{9}$$

Figure 1 presents the cost curves for inspection routine cost (I_{RC}), expected failure cost (E_{FC}) and total inspection routine cost ($TIRC$). The optimum intervals obtained was 8 months with I_{RC} = \$7,500.00, E_{FC} = \$8,790.01, $TIRC$ = \$16,290.0 and a mean reliability of 82,4 %. The computer algorithm used to develop the inspection cost optimization models was MatLab [MATHWORKS, 2003].

The influence of inspection routine cost (I_{RC}) and expected failure cost (E_{FC}) to determine optimal inspection interval can be observed in figure 2. As the ratio of cost U_{cc}/I_c decrease, i.e., the failure cost approximate to inspection cost, the optimum intervals increase in an exponential behavior.

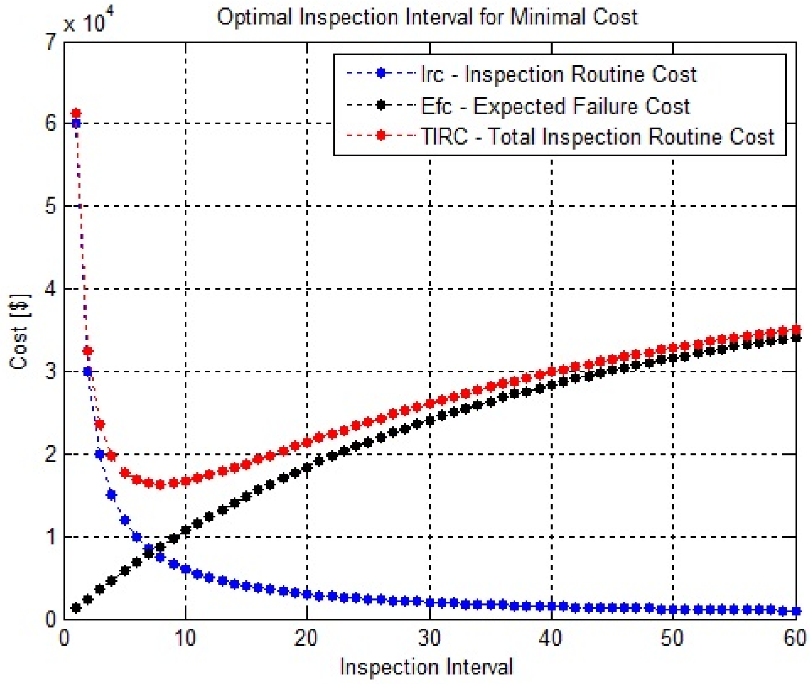

Figure 1 – Optimal inspection interval for minimum TIRC cost.

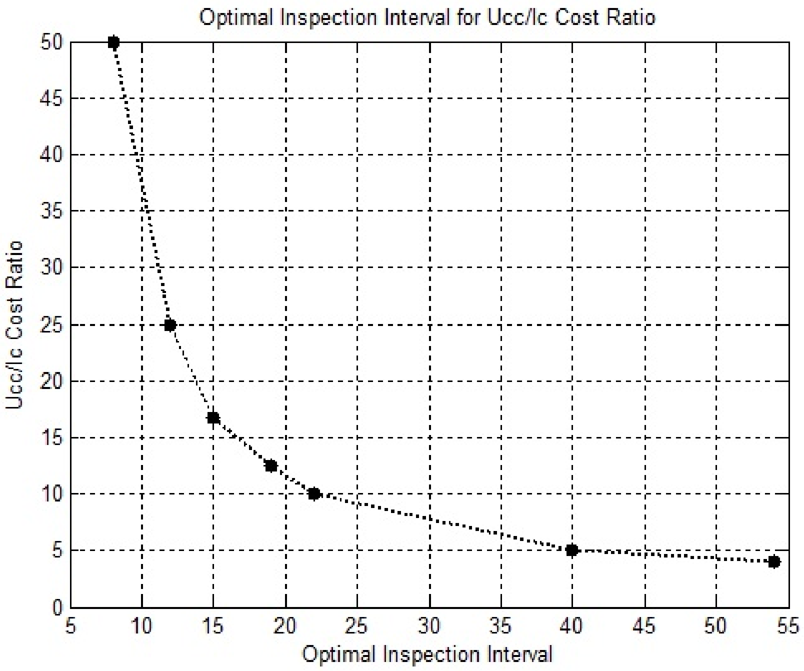

Figure 2 – Optimal inspection interval for failure/inspection cost ratio.

5. CONCLUSION

It was presented and developed in this paper a mathematical model to obtain an optimal inspection interval that minimizes total inspection route costs. The model uses constant failure rate commonly found in typical reliability database resources.

The simplicity of the proposed model allows maintenance professionals to set optimal inspection intervals avoiding apply Monte Carlo simulations or complex algorithms to lifetime data analysis with historical time to failures data.

For future works, it is planning to develop and apply the followings topics:

- develop the objective function to set optimal inspection interval that maximizes the availability of equipment or item studied;
- apply multi-objective functions to maximizes availability and minimize inspections costs at same time;
- compare optimal inspection intervals for constant failure rate with results using increasing failure rates with Monte Carlo simulations;
- publish worldwide the algorithms developed by using web API services.

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