A Probabilistic Model for the Design of Well Casings

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1. INTRODUCTION

It is nowadays widely accepted that optimization of the design of deep wells necessarily involves the quantification of uncertainties in geo-mechanical loads and in the strength of tubulars. This has led to the development of probabilistic models for design of well casings. This paper presents a framework for the probabilistic design of well casings.

This paper addresses the reliability of installed well casings during the perforation of the next phase, and reliability of the production tubing during production. The paper, and the models developed herein, are part of a more ambitious project [1,2], which will also consider a probabilistic model for geo-mechanical loads and their evolution in time; the modelling of strength degradation over time; the analysis of consequence scenario, the definition of acceptable risks and the optimization of well design balancing construction costs with acceptable risks. Such issues, however, are not addressed herein.

The various topics discussed herein are illustrated for a typical 19.000 ft well, following example calculations presented in Rahman & Chilingarian [3]. The pore pressure gradient and fracture gradient curves for the example well are shown in Figure 1 (left). The casing program to complete this well is presented in Figure 1 (right) and in Table 1. Details of drilling program, mud program and casing program, are presented in refs. [1-3]. Design conditions leading to maximum internal and external pressure loadings are also presented in refs. [1-3]. Details are not included herein due to space constraints.

This paper addresses the reliability of well casings considering the burst and collapse failure modes. Axial loading due to self-weight is also considered, as well as its effect on burst and collapse strengths. Other loading conditions such as annular pressure build-up are not considered herein, but will be addressed in future work.

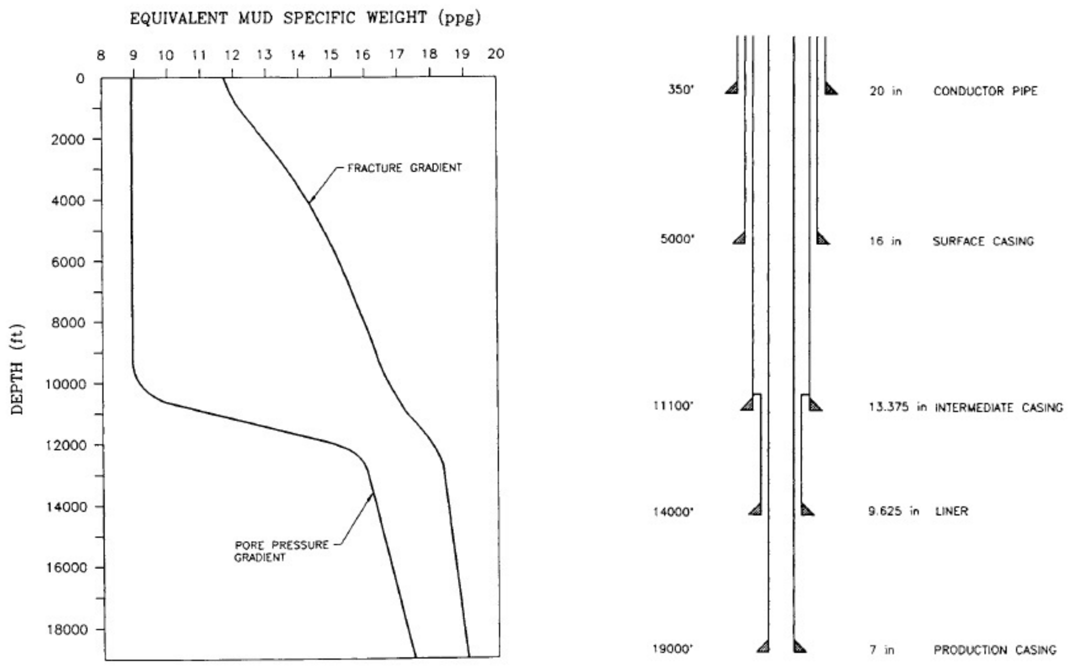

Figure 1 – Pore pressure and fracture gradients (left) and initial casing program (right) for typical 19.000 ft well [3].

Table 1: Casing program for typical 19.000 ft well [3].

Casing	Section	Depth	Class and weight (lb/ft)	Length (ft)
Surface	1	0 - 3550	L80, 84	3550
	2	3550 - 5000	K55, 109	1450
Intermediate	1	0 - 2000	P110, 98	2000
	2	2000-4000	L80, 98	2000
	3	4000 - 6400	P110, 85	2400
	4	6400 - 11100	P110, 98	4700
Liner	1	10500 - 12500	P110, 47	2000
	2	12500 - 14000	L80, 58	1500
Production	1	0 - 3000	W150, 38	3000
	2	3000 - 8000	MW155, 38	5000
	3	8000 - 16000	V150, 46	8000
	4	16000 - 19000	SOO155, 46	3000

2. RANDOM MODELS FOR TUBULAR STRENGTH

This section addresses random models for tubular strengths, considering the failure modes by burst and collapse, and the effects of axial stress. The ISO 10400:2011 code [4] presents a throughout analysis of model errors for strength prediction of OCTG tubulars.

In its appendix B, the ISO 10400:2011 code [4] compares the results of six models (Barlow, Von Misses, Klever-Stewart, Paslay, Moore, Naday) in predicting the burst strength of capped-end tubes submitted to internal pressure. Model error statistics are obtained for over 106 burst tests, and it is shown that one of the best predictions is obtained with the Klever-Stewart model. For this model, model error mean is very close to unity, and coefficient of variation (C.V.) is of 4.7% (Table 2). Thus, only the Klever-Stewart model [5] is considered herein.

In appendix F [4], eleven models for prediction of collapse strength of OCTG tubulars are compared. Based on over 3 thousand experimental results, obtained with tubulars of different manufacturers, one can conclude that Klever-Tamano is one of the best available models. The model error mean is close to unity and C.V. is 6.7% (Table 3). Therefore, only the Klever-Tamano collapse model [6] is considered herein.

Also in appendix F of ISO 10400:2011 [4], an ample evaluation of statistics for geometrical and material parameters of OCTG tubulars is presented. This includes measurements of yield stress, rupture stress, wall thickness, eccentricity and residual stresses. Three groups of statistics are presented: *ensemble*, *governing A* and *governing B*. Ensemble statistics are obtained for the whole database; governing A and B are sub-sets of the database, obtained for a particular class of tubulars, and which could lead to worst-case results in terms of reliability analysis. In this section, these three sets of statistics are propagated through the Klever-Stewart and Klever-Tamano models, in order to verify which set leads to the worst scenario in terms of well casing reliability.

2.1 Random burst strength by Klever-Stewart model

The Klever-Stewart burst model is based on four interconnected concepts: a) an equation based on equilibrium and plasticity for ductile rupture of the tube with known diameter and wall thickness; b) a strength reduction factor to account for wall thickness reductions originating in imperfections not detected during quality control; c) a criterion for minimum tenacity and d) a transition to necking under very large tensile loads. The Klever-Stewart burst strength equation is presented in references [1, 2, 4, 5].

In this section, uncertainties in rupture stress and wall thickness are propagated through the Klever-Stewart burst strength model. Statistics for *ensemble*, *governing A* and *governing B* properties (Table 2) are compared (but not all are shown). Seventeen tubulars [2] are considered in the analysis, but only results for L80 58 lb/ft are shown. Figure 3 shows, for the L80 58 lb/ft tubular, that the subset of statistics called *governing B* properties [4] leads to the smallest burst strengths. This result is also obtained for the set of 17 tubulars studied herein, revealing that the subset of *governing B* properties [4] leads to the worst case results in a reliability analysis.

Table 2: Random strength variables for burst pressure [4].

Variable		Symbol	Distribution	Mean	C.V. (%)
Model error, Klever-Stewart burst model		M_E	**Normal**	**1,0040**	**4,70**
Ensemble properties	Ultimate stress	f_u	Normal	1,1000	4,22
	Wall thickness	t	Normal	1,0069	2,59
Governing properties A	Ultimate stress	f_u	Normal	1,0938	3,33
	Wall thickness	t	Normal	0,9876	0,20
Governing properties B	**Ultimate stress**	f_u	**Normal**	**1,1000**	**4,22**
	Wall thickness	t	**Normal**	**0,985**	**0,10**

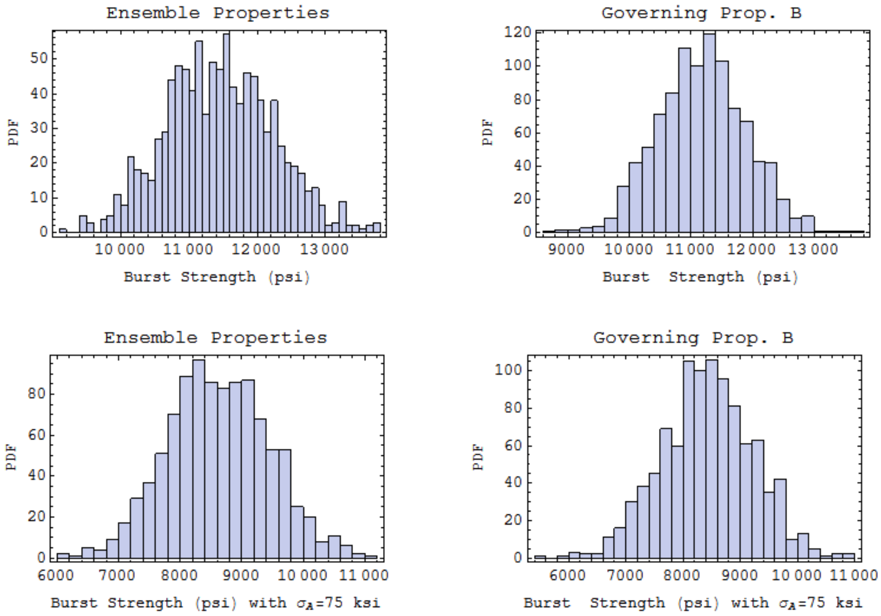

Figure 3: Histograms for 1000 samples of Klever-Stewart burst strengths, comparing *ensemble* and *governing B* properties, with (below) and without (above) axial stress [2].

2.2 Random collapse strength by Klever-Tamano model

The Klever-Tamano collapse model is based on a) an elastic collapse equation, applicable to perfect thin-walled tubes; b) a plastic collapse equation for perfect thick-walled tubes; c) a transition equation, from elastic to plastic collapse, applicable to tubes of any wall thickness and d) factors to account for imperfections, such as: ovality, residual stresses, eccentricity and form of stress-strain curves. The Klever-Tamano burst strength equation is presented in references [1, 2, 4, 6].

In this section, uncertainties in yield stress, wall thickness, ovality, eccentricity and residual stresses (Table 2) are propagated through the Klever-Tamano collapse strength model. Statistics for *ensemble*, *governing A* and *governing B* properties [4] are compared. Seventeen tubulars [2] are considered in the analysis, but only results for L80 58 lb/ft are shown. Figure 4 shows, for the L80 58 lb/ft tubular, that the subset of statistics called *governing B* properties [4] leads to the smallest collapse strengths. This result is also obtained for the set of 17 tubulars studied herein, revealing that the subset of *governing B* properties [4] leads to the worst case results in reliability analyses for casing collapse.

3. RANDOM LOAD MODELS

An extensive literature review performed by the authors [1,2] revealed that very little exists, in the openly available literature, on probabilistic modelling for well loads. Individual operators may have proprietary models which are not openly available. Surely, loading uncertainty plays a pivotal role in reliability of well casing. Pore pressures, for instance, are estimated based on correlation wells which are sometimes many kilometers away from the target well. Fracture gradients are also estimated by correlation, via lost circulation or intentional fracturing of nearby wells, or by fracture, leak-off or pressure integrity tests, all of which subject to significant uncertainty.

Table 2: Random strength variables for collapse pressure [4].

Variable		Symbol	Distribution	Mean	C.V. (%)
Model error, Klever-Tamano collapse		M_E	**Normal**	**0,9991**	**6,70**
Ensemble properties	Yield stress	f_y	Normal	1,1000	4,22
	Wall thickness	t	Normal	1,0069	2,59
	Ovality	*ov*	Weibull min.	0,217	54,1
	Eccentricity	*ec*	Weibull min.	3,924	66,1
	Residual stresses	*rs*	Normal	-0,237	33,2
Governing properties A	Yield stress	f_y	Normal	1,0938	3,33
	Wall thickness	t	Normal	0,9879	0,2
	Ovality	*ov*	Weibull min.	0,660	20,0
	Eccentricity	*ec*	Weibull min.	13,20	20,0
	Residual stresses	*rs*	Normal	-0,264	20,0
Governing properties B	**Yield stress**	f_y	**Normal**	**-**	**-**
	Wall thickness	t	**Normal**	**0,9850**	**0,1**
	Ovality	*ov*	**Weibull min.**	**0,795**	**10,0**
	Eccentricity	*ec*	**Weibull min.**	**15,90**	**10,0**
	Residual stresses	*rs*	**Normal**	**-0,318**	**10,0**

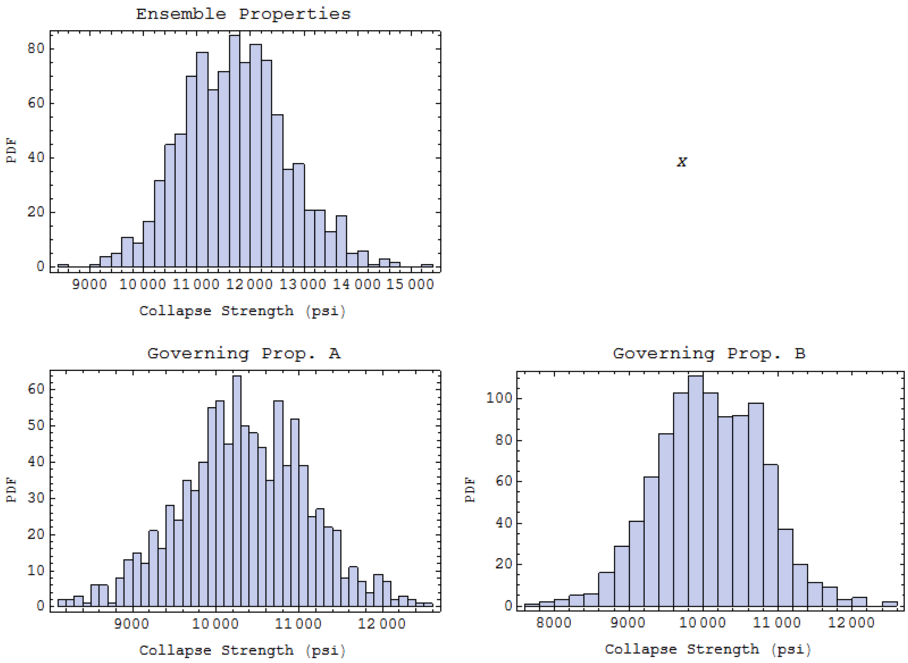

Figure 4: Histograms for 1000 samples of Klever-Tamano collapse strengths, comparing *ensemble* and *governing A* and *B* properties, without axial tension [2].

It is possible to perform a reliability analysis when part of the problem is not known, if sensitivity (or participation) factors are known or can be estimated. Sensitivity coefficients show the contribution of individual random variables towards calculated failure probabilities, in a First Order Reliability (FORM) setting. When random variables are grouped in loading (stress) and resistance (strength) variables, sensitivity coefficients reveal the contribution of loading and resistance variables:

$$(\alpha_R)^2 + (\alpha_S)^2 = 1 \quad (1)$$

Hence, when a reliability problem is solved considering only resistance uncertainties, as proposed here, the resistance reliability index is obtained (β_R). If the contributing factor α_R is known, than the problems reliability index is:

$$\beta = -\alpha_R \beta_R = -\alpha_S \beta_S \tag{2}$$

Figure 5 illustrates this issue. The figure represents a problem where the standard deviation of the load is significantly larger than the standard deviation of the resistance. This is typical of structural engineering problems. In this problem, both loading (S) and resistance (R) are normally distributed, with parameters R~N($\mu_R = 7, \sigma_R = 1$), S~N($\mu_S = 1, \sigma_R = 4$) (for illustration purposes – Figure 5 left). As the two variables are transformed to standard Gaussian space (Figure 5 right), one observes the relation between the reliability index β, the partial reliability indexes that would be obtained if only the resistance (β_R) or only the loading uncertainty (β_S) where considered, and the $\boldsymbol{\alpha}$ vector.

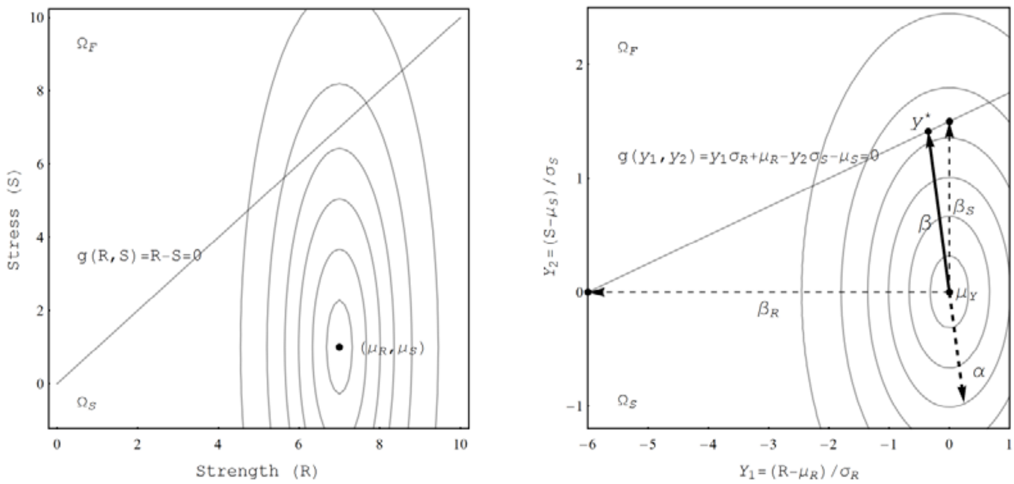

Figure 5: Separation of reliability problem in load and resistance parts, original design space (left), transformed standard Gaussian space (right).

4. CASING RELIABILITY

Evaluation of casing reliability is illustrated in this section, for the typical well described at the Introduction (Figure 1 and Table 1) and for the failure modes of burst and collapse. As explained in Section 3, only uncertainty in casing strength is considered. Since load uncertainty is not considered, all results in this section are for β_R. In our perception, the sensitivity of load uncertainty would be at least 50% ($\alpha_S \geq 0.5$). Hence, actual casing reliability could be significantly smaller than the values presented here.

The random variables affecting casing strength, for burst and collapse, are the *governing B* properties described in Section 2. The limit state function for burst or collapse failure is given by:

$$g(\boldsymbol{X}, h) = M_E p_R(\boldsymbol{X}, h) - p_S(h) \tag{3}$$

where $\boldsymbol{X}$ is a vector of random variables, M_E is the model error variable for the resistance (strength), h is the depth, p_R is the resistance pressure (for burst or collapse) and p_S is the loading pressure (internal pressure differential for burst and external pressure differential for collapse). Vector ***X*** contains the random variables affecting tube strength, as described in Section 2.

For all sections of the well (conductor, surface, intermediate, liner and production casing), the pressure differentials (internal and external pressures) are evaluated as a function of depth, following the loading cases described in [3]. Detailed calculations can be followed in refs. [2,3] and are omitted due to space constraints. Graphical results are presented below for the intermediate casing only.

4.1 Burst failure mode

Figure 6 shows the internal pressure, external pressure and pressure differential resulting from a gas kick at the well, during drilling of the final stage (production hole). These pressure differentials are inserted into Eq. (3), together with the resistances evaluated for the tubulars pre-selected for that phase (Table 1). Safety factors are evaluated for all depths. Critical points are identified as the points where safety factors are minimum. Failure probabilities are evaluated at critical points, using the First Order reliability method. Figure 7 shows the resistance reliability indexes evaluated at different depths. Table 4 shows safety factors and resistance reliability indexes evaluated at the critical points along the well, for all well phases.

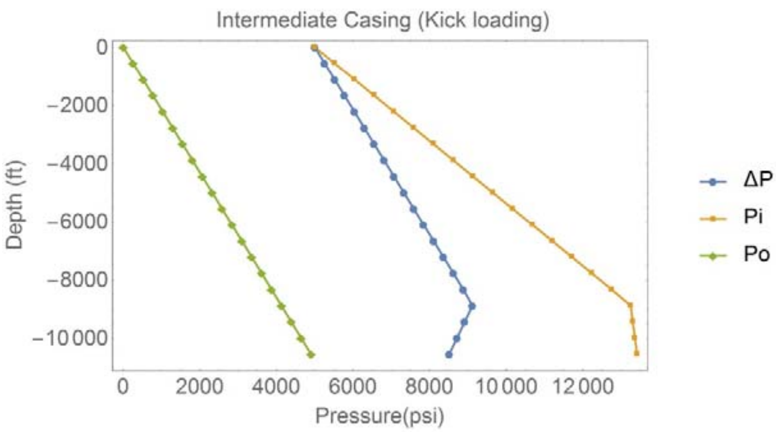

Figure 6: Burst pressure loading on intermediate casing due to a kick.

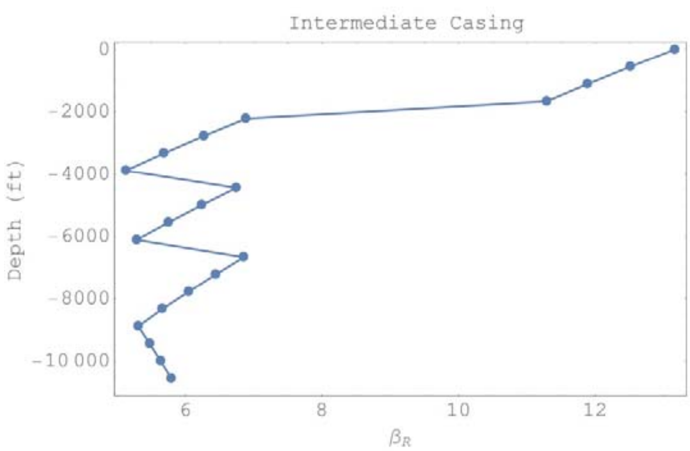

Figure 7: Resistance reliability index for burst failure of intermediate casing.

Table 4: Resistance reliability index and safety factors for burns failure at critical points of well.

Casing	Depth.(ft)	Pipe	Pext (psi)	Pint (psi)	Normal (psi)	S.F.	Beta
Surface Casing	0	84 lb/ft L80	500.	3967.6	18922.4	1.86146	7.23193
Surface Casing	3550.	109 lb/ft K55	1795.75	3967.6	6554.92	3.30233	8.74686
Intermediate Casing	3996.	98 lb/ft L80	1858.14	8719.48	23261.7	1.80762	5.01882
Intermediate Casing	6327.	85 lb/ft P110	2942.06	10889.2	15143.2	1.96446	5.10156
Intermediate Casing	8880.	98 lb/ft P110	4129.2	13247.2	6384.88	2.1149	5.30732
Intermediate Casing	1998.	98 lb/ft P110	929.07	6859.74	30111.	2.68048	10.9425
Liner	10500.	47 lb/ft P110	4882.5	13409.2	13380.1	2.18274	4.88242
Liner	12530.	58.4 lb/ft L80	5826.45	13612.2	6350.41	2.34021	4.39542
Production Casing	7980.	38 lb/ft MW155	3710.7	22669.6	46240.9	1.89126	6.216
Production Casing	2850.	38 lb/ft V150	1325.25	17894.6	64028.8	1.91139	7.78242
Production Casing	15960.	46 lb/ft V150	7421.4	30097.4	18687.4	2.07396	5.99267
Production Casing	18810.	46 lb/ft SO0155	8746.65	32750.1	8847.84	2.08616	5.66357

4.2 Collapse failure mode

Figure 8 shows the internal pressure, external pressure and pressure differential resulting from a lost circulation loading at the well, during drilling of the final stage (liner hole). Figure 9 shows the resistance reliability indexes for collapse evaluated at different depths. Table 5 shows safety factors and resistance reliability indexes evaluated at the critical points along the well, for collapse and for all well phases.

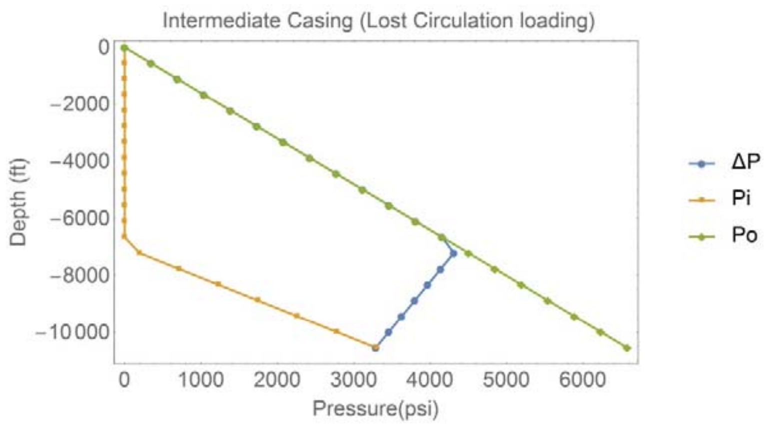

Figure 8: Collapse pressure loading on intermediate casing due to lost circulation.

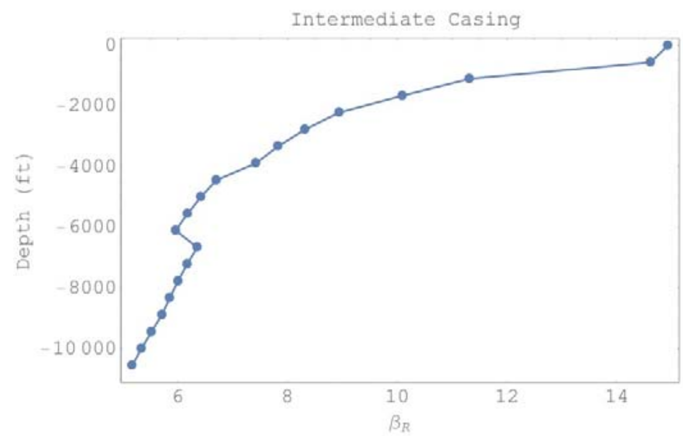

Figure 9: Resistance reliability index for collapse failure of intermediate casing.

Table 5: Resistance reliability index and safety factors for collapse failure at critical points of well.

Casing	Depth.(ft)	Pipe	Pext (psi)	Pint (psi)	Normal (psi)	S.F.	Beta
Surface Casing	3500.	84 lb/ft L80	1627.5	2184.	6729.11	2.34554	9.16532
Surface Casing	4950.	109 lb/ft K55	2301.75	3088.8	1729.2	2.65736	9.98681
Intermediate Casing	6327.	85 lb/ft P110	3948.05	0	15143.2	1.41673	5.46421
Intermediate Casing	6993.	98 lb/ft P110	4363.63	0	12853.7	1.92238	8.0724
Intermediate Casing	3996.	98 lb/ft L80	2493.5	0	23261.7	2.83461	10.305
Intermediate Casing	1998.	98 lb/ft P110	1246.75	0	30111.	6.68024	12.9526
Liner	10535.	47 lb/ft P110	9203.38	3284.78	13258.9	1.69741	2.28331
Liner	12530.	58.4 lb/ft L80	10946.2	5141.72	6350.41	2.46667	4.37769
Production Casing	18810.	46 lb/ft SO0155	17508.3	0	8847.84	1.73223	7.21357
Production Casing	15960.	46 lb/ft V150	14855.6	0	18687.4	1.98294	8.18054
Production Casing	7980.	38 lb/ft MW155	7427.78	0	46240.9	2.80148	10.2434
Production Casing	2850.	38 lb/ft V150	2652.78	0	64028.8	7.53952	13.1855

4.3 Discussion on system reliability and design optimization

The different critical points shown in Figures 7 and 9, and in Tables 4 and 5, represent a series system with multiple possible failure locations, and two main failure modes. The failure modes for burst and collapse can be assumed to be independent and mutually exclusive, since they arise from very different loading scenarios.

Failures at different locations for the same mode are correlated events, which characterize a series system. Failure probabilities for a series system are always dominated by the weakest links, especially for highly correlated events. Having some of the links (parts of the well) with significantly higher reliability, for instance, does not impact on system reliability. Hence, large reliability index indicate sections of the casing that could be down-graded, saving resources in well completion, without impacting well safety.

In Figure 7 and Table 4, and also in Figure 9 and Table 5, one observes significant differences in terms of reliability indexes along the depth, for a single phase and for different phases. For burst of the intermediate casing, in particular (Figure 7), most sections have reliability indexes around five. Between zero and two thousand feet, however, reliability indexes are significantly higher, above ten. For collapse failure (Figure 9), the reliability index is around 5 to 6 at greater depths, but significantly larger at shallower depths. This shows a situation where significant savings could be achieved by down-grading the tubes at shallower depths of the intermediate casing, saving resources without compromising safety.

5. CONCLUDING REMARKS

In this paper some aspects of reliability analysis and probabilistic design of well casings have been discussed. Random models for casing strength are already well developed, and have already been included in design codes [4]. These models account for variabilities in geometrical and material properties. Data on model errors for burst and collapse strengths have been obtained, and represent a significant source of uncertainties in tubular resistance. Model errors on tube strength have mainly been obtained under zero axial stress; hence any model errors arising from axial stress have not been properly evaluated.

Random models for loading are not yet available in the openly published literature. Loading uncertainties play a significant role in failure probabilities for civil engineering structures. Hence, one should avoid the naivety of assuming that resistance reliability indexes represent casing reliability indexes; their do not! Casing reliability indexes are assumed to be significantly smaller than reliability indexes computed considering only resistance uncertainties.

Computation of reliability indexes have been shown in this paper, for the case of a typical well. One observes significant differences in reliability indexes computed at different critical points, at the same phase or at different phases of a well. Since the different critical locations along a well constitute a series system, whose failure is controlled by the weakest link, sections with significantly larger reliability indexes could be down-graded, contribution to cost reduction without compromising well safety.

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