

## Comparison of Two Models for Damage Accumulation in Simulations of System Performance

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### 1. INTRODUCTION

As methods and tools for simulation of system performance become more powerful, it is becoming more feasible to model underlying physical causes of system failure more explicitly, with a view to thinking more clearly about how to achieve, and then demonstrate, satisfaction of a reliability requirement. An important analysis method for this is simulating an ensemble of time histories that collectively span the issue space of interest: specific aleatory and epistemic uncertainties are sampled in such a way that the ensemble of simulation results can be interpreted usefully. For example, pdfs of key Figures of Merit (FOMs) can be obtained.

For generations, it has been practical (and widely done, e.g., [1]) to analyze the reliability and availability of complex systems in this way, based on classical reliability parameters for the system components, provided that the scenario-specific physics does not dynamically change component behavior. If scenario-specific physics does dynamically alter component behavior, the problem is still doable [2], but appreciably more complex than simulating in either domain (physics or reliability) alone, especially if the reliability aspects are addressed by querying, at each time step, whether a component state change has occurred.

This paper is concerned with two simulation-based approaches to modeling system-level performance when component degradation depends on scenario-specific conditions:

1. Heartbeat model [3]: The accumulation of damage to components is determined mechanistically by scenario physics, with component failure occurring when an aleatory threshold for component failure is reached. This model was formulated specifically to allow incorporation of reliability characteristics into the simulation of physics without significantly degrading the efficiency of the simulation.
2. Diffusion model [4, 5]: The accumulation of damage to components is modeled as a diffusion process, with failure occurring when accumulated damage crosses a fixed threshold.

Both approaches can be applied within a framework in which the variability in component failure time (and, of course, the effect at the system level) is assessed through post-processing of numerous time histories, representing an appropriate sampling of the relevant issue space.

One purpose of the present paper is to resolve some confusion in the existing literature by suggesting that these are not best viewed as models of the same thing. Rather, based on the “load vs. capacity” idea reviewed briefly below, the diffusion model is best viewed as a model for load, and the heartbeat model is best viewed as a model for capacity. They can, of course, be used together, but the diffusion model seems oversimplified for this class of applications, except perhaps as a source of noise in the phenomenological model within which a heartbeat model might be used.

Section 2 very briefly reviews the concept of load-vs.-capacity models. Section 3 recaps the heartbeat model and the diffusion model. Having briefly discussed the two models, the paper then illustrates their combination, addressing:

- aleatory variability of component damage tolerance,
- the effect of scenario physics on the rate of damage accumulation,
- random (aleatory) variability in the rate of damage accumulation.

## 2. LOAD-VS.-CAPACITY MODELS

Many discussions of system reliability nowadays are carried out in terms of the load-vs.-capacity idea illustrated in Figure 1.

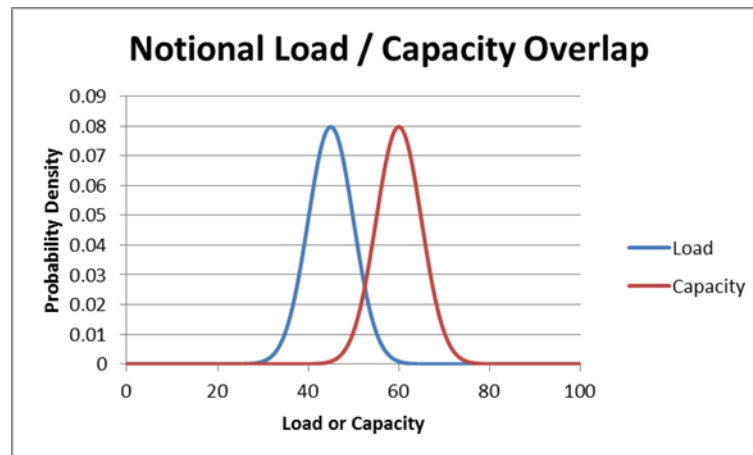


Figure 1. Overlap of probability density function (pdf) of “load” and pdf of “capacity”

The idea behind Figure 1 is that if a component has the “capacity” to withstand the “load” imposed on it in a particular situation, it will not fail; but if load exceeds capacity, it does fail. In this description, “load” and “capacity” can refer to many different things: mechanical stress, temperature, and so on. In many classes of scenarios of interest, these effects are either variable or uncertain, and need to be analyzed in the context of an ensemble of simulations chosen to sample a relevant issue space in an appropriate way. In the figure, most of the load pdf is to the left of most of the capacity pdf, so it is evident by inspection that, assuming “load” and “capacity” are independent in the scenario of interest, and have the distributions shown, the component will usually succeed, but has a nontrivial failure probability.

Simulation models can, of course, address the variability and uncertainty in both the load and capacity constructs.

## 3. HEARTBEAT MODEL

In principle, simulating physics and component reliability together has been doable for many years. As noted above, if the scenario physics does not affect component behavior, one can sample component failure time *a priori*, and factor this change of system state into the physics simulation as part of the input specification. However, if the time-history-specific physics affects component behavior, and if we need to address uncertainty and variability in component behavior, then the above input-specification approach is inadequate. One could query the status of component operability at each time step, based on a failure-time distribution modified by scenario physics, but this has the potential to slow down the simulation quite significantly, by requiring the simulation to take smaller time steps than the physics simulation alone could tolerate, especially if there are time-history-specific influences on component behavior. Accordingly, [3] presented an approach within which component failure is anticipated in a way that permits the physics-allowable time step to be used nearly all the time, and reduced only when we know a component state transition is about to occur.

The authors of [3] refer to this as the “heartbeat model,” based on the old idea that people are born with a certain number of heartbeats, and die when that number is used up (if not before, from other causes). For a fixed number of heartbeats, variation in the rate of using up heartbeats induces a variation in the time of death; and, of course, each individual’s initial allocation of heartbeats is different.

Analogously, we can interpret a probability density function (pdf) on component failure time as a pdf on component damage tolerance. If the influences on component state are independent of time, nothing has changed; but if the influences on the component are time-dependent, the pdf changes shape somewhat if plotted as a function of time. Within this approach, in order to initialize the simulation of a time history, we choose a component from this distribution, and thereby determine the level of usage (or damage) at which this component will fail in that time history; during the simulation of that time history, the rate of damage accumulation may vary, just as heart rate varies as a function of current activity and general health. Tracking the accumulated damage as the simulation proceeds, and knowing at each simulation time step how close accumulated damage is to the time-history-specific component failure threshold, we can reduce the time step as necessary to keep the simulation numerically accurate, and then increase the time step again once the simulation is safely past the change in component state.

The need to take account of environmental influences on component reliability has long been recognized, and guidance for doing so has long been promulgated in certain fields. For example, MIL-HDBK-217 [6] discusses situation-specific failure rate models such as the following:

$$\lambda_p = \lambda_b \pi_1 \pi_2 \pi_3 \dots \quad (1)$$

where

$\lambda_p$  is the situation-specific failure rate associated with a “part,” such as an electronic component,

$\lambda_b$  is the base failure rate,

the  $\pi$  factors modify the base failure rate for the category of environmental application and other parameters that affect the part reliability, such as temperature in the situation-specific operating environment.

The point of the heartbeat model was not to supplant or neglect this sort of guidance, but rather to support its incorporation into simulations addressing physics and component reliability together, comprehensively exploring the relevant issue space of aleatory and epistemic uncertainties, without having the reliability model limit the simulation time step except when significant transitions are in progress. In the real world, when we choose a component from Laplace’s urn, we do not know when it will fail; all we know is the failure time distribution of the components in the urn (and usually we are uncertain about that).

The literature of accelerated life testing relates component reliability characteristics to the imposition of stressors, such as elevated temperature or mechanical loading. See [7] among many other references on this general topic (“cumulative damage”), and [8] for an example application in a thermal-hydraulics code.

Some workers model component failure as if it were a stochastic process, determined by a time-dependent failure rate but essentially capable of occurring at any instant. The heartbeat model acknowledges that we do not know when a failure will occur, but regards failure as a mechanistic process whose outcome is determined by the initial conditions (including the microscopic characteristics of the system components) and the physics and chemistry of the evolution of the scenario. Sometimes the failure times will collectively obey a classical reliability distribution, and sometimes, the scenario-specific stressors will either speed up or slow down the accumulation of damage; within the heartbeat model, then, components are representative of the same pdf that they obeyed before, but the pdf is now over component damage, not necessarily time.

Accordingly, within the present framework, we treat the problem of the uncertainty in failure thresholds (among others) by performing an ensemble of simulations, comprising a representative sample of the distribution of component failure times. Some reliability models treat component failure as essentially stochastic: component failure can happen any time. The heartbeat model puts the aleatory part of the problem into the initialization of each time history in the simulation, when we select the component from Laplace’s “urn” containing nominally identical components whose failure times are represented by one of the classical reliability functions (e.g., Weibull).

#### 4. DIFFUSION MODEL

This model [4] was recently used in a benchmark exercise [5] calling for a simulation-based analysis of the mission success probability for a space propulsion system on a long (~78000-hour) mission to distant planets. The purpose of the exercise was to highlight the relative advantages and disadvantages of different

simulation-based approaches to the problem. Among the components to be modeled were the fuel distribution lines. According to the problem statement [5],

Distribution lines: the damage accumulation  $D(t)$  of the distribution lines can be modeled as a Gaussian random walk (Brownian motion) having mean value (drift) equal to 1 [per unit (1 hour) time step] and sigma equal to 0.4. Distribution lines fail when  $D(t) = 80,000$ ; when this happens the mission is lost.

Since the average damage accumulation is one unit per hour, the mission is 78000 hours, and the failure threshold is 80000, the average damage accumulation will not fail the lines; but since the given value of sigma is greater than 0, there is some scatter around the average damage accumulation at 78000 hours, and the tail of that distribution exceeds 80000 a small fraction of the time.

## 5. DISCUSSION AND CONCLUSIONS

Each of the two models introduces uncertainty into the outcome of a simulation, but with different underlying causes at work.

1. The original heartbeat model is anchored in the assumptions of classical component reliability modeling and accelerated life testing, and formulated so as to work efficiently in the context of simulations of complex time-dependent phenomenology. As a time history is initialized, an individual component's capacity is sampled from a classical distribution, and in the simulation, "damage" is accrued depending on current environmental conditions in the simulation. In order to determine the time of failure in the simulation, accrued damage is compared with this capacity. Every individual time history is simulated mechanistically (there is nothing stochastic in any individual time history), conditional on sampled values of epistemically uncertain variables, and on the sampled aleatory uncertainties.
2. In the stochastic model approach, damage within a time step is quantified based on a random variable sampled at each time step. Unless one believes in the indeterminacy of component physics, this is not a natural description of inherent component behavior. But it could be a reasonable description of a fluctuating external influence – a stressor, or a "load" - on a system. In the parlance of the classical load / capacity diagram, where the distribution of loads and the distribution of capacities are plotted together, this corresponds most naturally to the load distribution, and the diffusive character of it corresponds to a particular kind of uncertainty about the scenario phenomenology.

In the case of the benchmark problem mentioned above, although the model for damage accumulation has a small noise contribution, and the rate of damage accumulation depends on time, the distribution of time to failure can be calculated outside of the much more elaborate simulation of the active-component failure modes, as follows. One samples a number between 0 and 1 to determine a component damage threshold, and applies this threshold to the plot of damage vs. time to obtain the distribution of times at which the failure threshold is reached. The only reason to have included this diffusion model for damage accumulation in the full simulation (instead of quantifying it separately) is that in some time histories, other failure modes will end the mission before this mode's failure time is reached; quantifying failure in separate models and summing the contributions is accurate only within the domain of validity of the rare-event approximation.

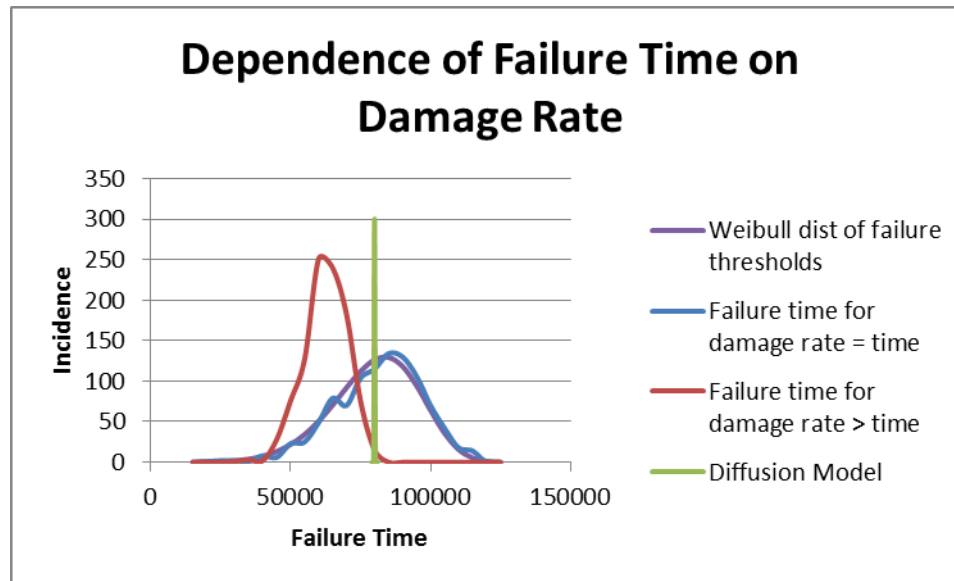


Figure 2. Dependence of Failure Time on Damage Rate

Figure 2 compares essential characteristics of the two models. A fairly strongly peaked Weibull distribution ( $\alpha = 6$ ,  $\beta = 86230$ , mean  $\sim 80000$ ) was used to represent the heartbeat model. The violet curve shows this peak. The blue curve represents failure times obtained from 1000 simple simulations in which failure “time” was sampled from this Weibull, and damage accrued at one unit per time interval. By construction, the blue curve approximates the violet one, with some kinks resulting from statistical fluctuation in the finite number of samples. The red curve is derived in the same way as the blue curve, except that now, damage accumulates more quickly for part of the simulation time. This visually illustrates the general idea that in many cases, the distribution of failure time influenced by varying damage rates, will be simply a shift and/or a distortion of the distribution used to generate damage thresholds.

The final “curve” on Figure 2, essentially a delta function, is derived from a diffusion model, using parameters similar to those in [5]. In that work, damage within a given time step was modeled as a locally constant mean value plus a fluctuating component having zero mean and constant variance. Clearly, one knows the mean of the resulting damage accumulation (averaged over time histories) as a function of time *a priori*. The variance of the distribution of failure times is, however, small. The square root of the variance goes as the square root of the number of time steps, in this case 80000; the relative width of the resulting distribution (say,  $3\sigma / (\text{mean})$ ) correspondingly goes as the reciprocal of the square root of the number of time steps. As a result, in Figure 2, the width of the failure time distribution using [5]’s parameters for the diffusion model with a per-step sigma of 0.4 cannot be seen.

The uncertainty associated with the diffusion model in Figure 2 would be considered extremely small for a component failure time distribution. In order to make it larger, we would need to use an enormously larger fluctuating component in the damage accumulation model, or work with a much smaller number of much larger time steps, or perhaps reduce the mean damage per time step and increase the fluctuating component. Those tactics do not seem particularly natural for this application. But the benchmark model formulation [5] using the diffusion model did its job: it challenged the methodologists.

Since the two models are naturally applied to distinct kinds of uncertainty, it is natural to contemplate applications that combine the two. “Capacity” can be determined within a heartbeat model, and “load” can, if appropriate, be modelled with a stochastic contribution. Within such a combination, time-stepping in the simulation of time histories can still be controlled by the physics and not by the pseudostochastic interpretation of component reliability models, except when the accrued damage is close to the capacity threshold, where “closeness” is measured in terms of the capability of the diffusion process to push accumulated damage over the threshold in the next few time steps.

Unfortunately, the parameters of many models for component reliability are known only very roughly. This is true even for simple (one-parameter) models. For this reason, point estimates of reliability cannot be

regarded as accurate. The heartbeat model simultaneously offers a way of partially addressing this problem, and exacerbates it, by adding to the number of parameters that need to be characterized.

## 6. ACKNOWLEDGMENT

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