FATIGUE ASSESSMENT BASED ON STRUCTURAL RELIABILITY FOR FLEXIBLE RISER ARMOUR WIRES

Vinícius Ribeiro Machado da Silva
Federal University of Rio de Janeiro/COPPE
Brazil

Luis V. S. Sagrilo
Federal University of Rio de Janeiro /COPPE
Brazil

Mario Alfredo Vignoles
Consultant Engineering
Brazil

1. ABSTRACT

Different solutions with different configurations, using rigid and flexible pipes, has been adopted by the operators to realize the oil and gas exploitation in ultra-deep water regions. The risk of failure can increase with each new challenge, and to facilitate its quantification, the using of methodologies based on structural reliability are needed for the application within the design of flexible pipes. This paper has the objective to do a preliminary analysis, under the structural reliability view, of the critical failure mode for the design of flexible pipes faced in Brazil, represented by the fatigue phenomenon on metallic layers. The study refers to the evaluation of the tensile armors, located at the bend stiffener region, for two flexible risers' configurations: free hanging and lazy-wave. Both consider the scenario of ultra-deep water, subjected to the same external loads. The methodology is based on that presented in Ref. [2], including new random variables. Sensitivity analyses are also performed for the variables that most contributes to the quantified probability of failure. Calibrated safety factors (SF) are obtained for different target failure probabilities.

2. INTRODUCTION

The current design philosophy of flexible risers are based on the use of deterministic variables that drive the structure's failure modes evaluation, which in most of the time need to be defined in a conservative way, due to a superposition of conservatisms. Structural reliability is a type of analysis that enables the engineer to consider the variable uncertainties in the structure design, carrying with it strong statistic concepts. This kind of analysis is already standardized for rigid risers [3,4], but its application to flexible pipes is very promising. Thus, a methodology based on this type of analysis was proposed in Ref. [2], in order to evaluate the fatigue failure of flexible riser's tension armours. The methodology permits to obtain the structure's probability of failure and enables the calibration of design safety factors according to a target probability of failure as an alternative of using those conservative indicated in the standards.

Two examples of application, of the mentioned methodology, are presented considering two different types of flexible riser configurations, free hanging and lazy-wave, both facing the same load conditions scenario. Statistical behavior for the set of random variables are taken from what have been already published in the literature, and two new random variables are proposed: one related to the simulation length of the global analysis, and the other is related to the end-fitting mounting that can influence the tension balance on the armour wires.

3. STRUCTURAL RELIABILITY FATIGUE METHODOLOGY

Structural reliability analysis aims at identifying the safety margin of a structure by quantifying the probability of a failure mode to happen, and this measure is called probability of failure. For this quantification, loads and resistances are taken as random variables characterized by their joint probability density function.

In order to assess the probability of failure, a failure function must be developed, and it is commonly designated as G(X). The vector X comprises all the random variables involved in the problem, and dictates whether the limit state is violated ($G(X) \leq 0$) or not ($G(X) > 0$). $G(X) = 0$ is defined as the failure surface. As can be seen in [5] the failure probability is defined by the following multi-dimensional integral:

$$p_f = P[G(X) \leq 0] = \int \dots \int_{G(X) \leq 0} f_X(x) dx \tag{1}$$

where $f_X(x)$ is the joint probability density function of all random variables X. Numerical and analytical methods can be applied to solve this integral. Monte Carlo Simulation (MCS), a numerical technique can be used together with variance reduction methods, as the Importance Sampling (MCSIS), to solve the problem with less iterations. The so-called First Order Reliability Method (FORM) is an analytical technique that approximate the failure probability result, providing also as an output the importance factors. These factors indicate the relative importance of each random variable on the computed failure probability.

The fatigue methodology [2] is based on the most used fatigue failure criteria by engineers in structural design: Palmgren-Miner's summation rule for fatigue damage associated to S-N curves. The fatigue design criteria is as follows:

$$\frac{T_{Ref} K_D}{\sum_{i=1}^{N} n_i (S_i)^m} \frac{\Delta}{SF} = \frac{T_{Ref}}{D_{T_{Ref}}} \frac{\Delta}{SF} = \frac{FL}{SF} > T_{oper} \tag{2}$$

where K_D and m are the design S-N curve parameters, S_i are the stress cycles ranges identified in a reference period T_{Ref} in years and $D_{T_{Ref}}$ is the fatigue damage accumulated in the period T_{Ref}. Δ is the allowable limit of accumulated fatigue damage (usually $\Delta = 1$ in deterministic fatigue analysis), FL is the computed fatigue life, T_{oper} is the structure design lifetime in years and SF is a safety factor. In this work, the mean stress effect is considered by means of the Gerber approach [6].

By considering uncertainties in the fatigue analysis, two limit state functions can be formulated to evaluate the fatigue probability of failure from installation to T_{oper} and to $\left(T_{oper}\text{-}1\right)$ as [2]:

$$G_1(\mathbf{X}) = X_1 \text{-} D_{T_{ref},BaseCase} \times \frac{T_{oper}}{T_{ref}} \times f(X_2, X_3, \dots, X_n) \tag{3}$$

$$G_2(\mathbf{X}) = X_1 \text{-} D_{T_{ref},BaseCase} \times \left(\frac{T_{oper}\text{-}1}{T_{ref}}\right) \times f(X_2, X_3, \dots, X_n) \tag{4}$$

where X_1 is a random variable representing the Miner's rule limit of accumulated fatigue damage and $(X_2, X_3, \dots, X_n)$ is a set of other random variables considered in the analysis. $D_{T_{ref},BaseCase}$ is the fatigue damage accumulated in a period T_{Ref} considering a specific set of values for the random variables $\mathbf{X}=\mathbf{X}_{BaseCase}$ and $f(X_2, X_3, \dots, X_n)$, where

$$f(X_2, X_3, \cdots, X_n) = \frac{D_{T_{Ref}}(X_2, X_3, \cdots, X_n)}{D_{T_{Ref},BaseCase}} \tag{5}$$

is a normalized surface response function. In the present work this function has been represented as:

$$f(X_2, X_3, \cdots, X_n) = f_2(X_2) f_3(X_3) \cdots f_n(X_n) \tag{6}$$

For a given random variable X_i, the corresponding $f_i(X_i)$ is represented by a second order polynomial that is fitted by using at least three values of X_i. One is that used to evaluate the $D_{T_{Ref},BaseCase}$, i.e. $X_i^{BaseCase}$, and the two others are $X_i^{BaseCase} \pm \kappa\sigma_{X_i}$, where σ_{X_i} is the standard deviation of X_i and, usually, $\kappa = 2$. For each of these analyses, the other variables are kept at their values defined in the vector $\mathbf{X}_{BaseCase}$. Figure 1 illustrates this fitting.

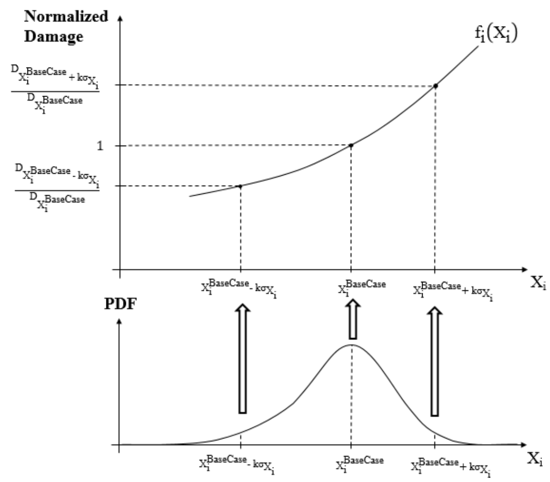

Figure 1 – Normalized function for X_i

Safety classes are defined [2] according to the riser´s function, in order to define acceptable levels of failure probability. The level is defined for the last year of operational riser´s life, i.e. the probability of fatigue failure during this year considering it is safe in the beginning of the year. This conditional probability can be approximated as [2]

$$p_f = p_{f1} - p_{f2} \tag{7}$$

where $p_{f1} = P[G_1(\mathbf{X}) \leq 0]$ and $p_{f2} = P[G_2(\mathbf{X}) \leq 0]$. Both probabilities can be evaluated by MCS or FORM.

Also within this methodology, safety factors can be calibrated with some algebraic manipulations of the limit state functions (3) and (4). Refs. [7] and [8] present the steps for the SF inclusion into these equations, which are presented as:

$$G_1(\mathbf{X}) = X_1 - \frac{1}{SF} \times f(X_2, X_3, \ldots, X_n) \tag{8}$$

$$G_2(\mathbf{X}) = X_1 - \frac{1}{SF} \times \left(\frac{T_{oper} - 1}{T_{oper}}\right) \times f(X_2, X_3, \ldots, X_n) \tag{9}$$

4. CASE STUDIES

Two 6” flexible risers applications, composed by the same structures, connected to a spread moored FPSO in a water depth of 2140m, are considered to be working in a region with similar features as those found in Brazil’s offshore pre-salt area, for a design life of 30 years. Case 1 considers the use of a free-hanging configuration and Case 2 a lazy-wave configuration for the flexible pipe (see Figure 2).

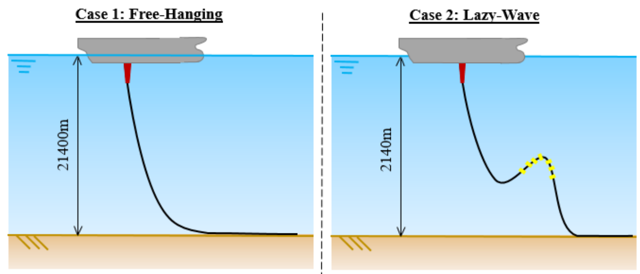

Figure 2 – Flexible risers configurations

The current is represented by a single profile, with varying velocity and directionality all along the water depth. The wind is considered to be integrated to the static offsets imposed to the FPSO, as currently assumed in the industry. Irregular wave time domain global analyses are performed with DEEPLINES [9] to obtain the global load effects along the riser considering a total scatter diagram of 88 sea-states coming from eight directions. A local analysis based on the analytical model [10,11,12] is employed to obtain the stress time histories for the armour wires. Rainflow cycle counting method [13] is used to identify the mean and alternate stresses and the associated number of cycles for each sea state. A linear S-N curve with parameters $\log K_D = 15.36$ and $m = 4$ is considered for the fatigue calculations.

Structural reliability analysis considers Monte Carlo Simulation with Importance Sampling (MCSIS) and FORM to obtain the fatigue probability of the armour wires at the bend stiffener region. Safety factors are also calibrated for three target failure probability levels 10^{-3}, $10^{-4}$and 10^{-5}, which represent respectively the low, normal and high safety classes [4].

The analysis sequence scheme is shown in Figure 3.

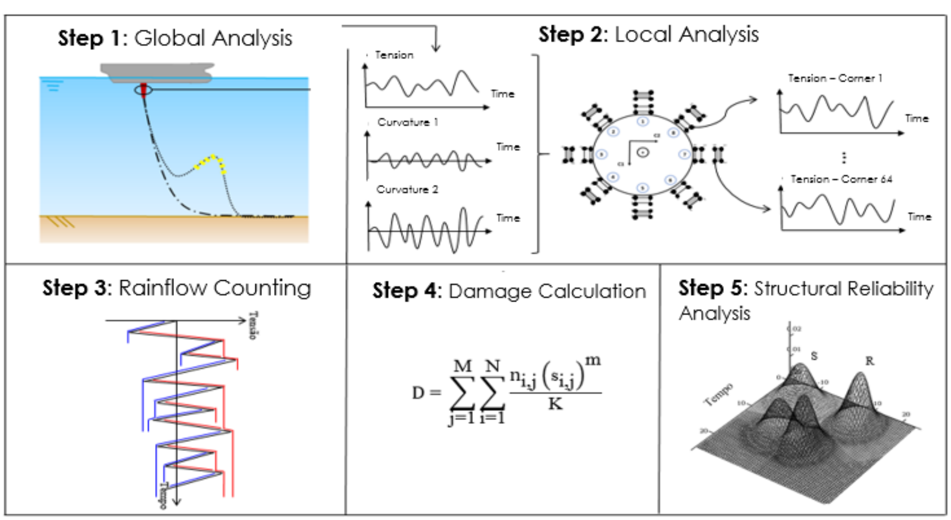

Figure 3 – Analysis sequence scheme [8]

The 13 variables shown in Table 1 were employed to characterize the randomness of the analysis in both cases studied. The statistical characterization is based on the values found within the literature [2,7], except for the following two now included: Irregular Wave Statistical Uncertainty (X_5) and Armour Wires Tension Distribution (X_6). The former represents the statistical uncertainty associated to the limited simulation length used within global analyses [8] and the later accounts for non-uniform distribution of tensions in the armour layer due to end fitting mounting process [1]. All random variables are considered as statistically independent.

Table 1 – Random variables for the case studies

Random Variable		Distribution Type	Base Case	Mean Value	Std. Dev.
X_1	Palmgren-Miner's rule limit damage	Lognormal	1	1	0.3
X_2	Drag Coefficient	Lognormal	1	1	0.2
X_3	Static Offset	Lognormal	1	1	0.08
X_4	Floater RAO	Lognormal	1	1	0.05
X_5	Irregular Wave Statistical Uncertainty	Lognormal	1	0.99*/ 1.02**	0.047*/ 0.036**
X_6	Armour Wires Tension Distribution	Lognormal	1	1***	0.069***
X_7	Internal Pressure	Normal	1	1	0.15
X_9	Friction Coeficient	Uniform	1.14	1	Limits Min: 0.86 Max: 1.14
X_{10}	Ultimate Stress	Lognormal	1	1	0.08
X_{11}	Armour Wire Settlement Angle	Lognormal	1	1	0.03
X_{12}	Global Analysis Method Uncertainty	Normal	1	1	0.05
X_{13}	Local Analysis Method Uncertainty	Normal	1	0.9	0.15

*Free-Hanging configuration / 3800s of simulation time
**Lazy-Wave configuration / 3800s of simulation time
***Region supposed to be far from the end fitting and less influenced by the mounting process

The use of regular wave approach to launch time domain global analysis is a fast and conservative manner to represent the wave elevation, and is commonly used by the industry. Irregular or stochastic wave generation try to represent better the randomness' nature that occurs in the reality. This last approach is not employed in the day-life of the flexible riser design engineer because of its high computational cost, once a 3 hour simulation length is often adopted (minimum required to guarantee sufficient statistical stability to provide less scattered results). The randomness of the fatigue damage results (X_5) associated to simulation length (sample size) is expected to be larger for shorter time domain simulations. Figure 4 shows the coefficient of variation (CoV) of the fatigue damage as a function of simulation length for both cases. These results were obtained using different realizations of the wave elevation process and making use of the bootstrap resampling technique to obtain the 99% confidence curve for the statistical estimator [8].

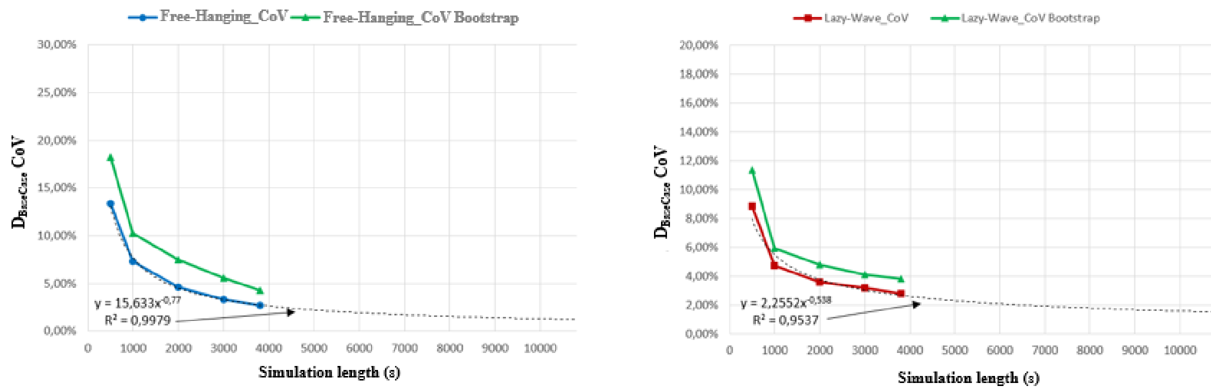

Figure 4 – Fatigue damage CoV as a function of simulation length (left: Free-Hanging; right: Lazy-Wave)

The end fitting mounting process inserts another source of uncertainty to the analysis. Depending on the process adopted, each tensile wire can be rearranged in a different position compared to the ideally designed in the project. This is associated to the level of disturbance that the wires are undertaken, leading to a non-uniform stress distribution on the armour wire's layers when they are sought. Ref. [1] handles with this phenomenon and assess the stress distribution on the external armour wires, in a region close to the end fitting, for 2 different mounting processes. This effect can be minimized, or dissipated, in locations distant from the end-fittings. In order to account for this phenomenon, this work include X_6 to the set of random variables, assuming the statistical variability characteristics described in [1] to be the same for the internal armour wires of Cases 1 and 2.

5. STRUCTURAL RELIABILITY ANALYSIS RESULTS

Finishing all the analysis process steps (Figure 3), the failure probabilities are quantified for the Free-hanging and Lazy-Wave configurations. Figure 5 shows the evolution of the tensile armour wire probability to experience a fatigue failure in the last operational year for different design lives (T_{oper}).

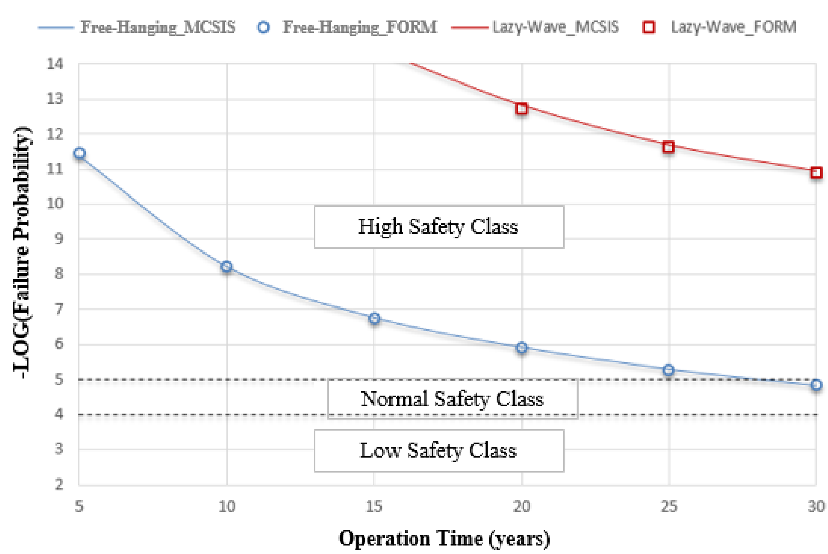

Figure 5 – P_f vs. T_{oper} for Cases 1 and 2

MCSIS analysis results shows that for the free-hanging configuration, considering an operational design life (T_{oper}).of 30-yr, it has a p_f = 1.45x10^{-5}, attending the criteria of a normal safety class, while the lazy-wave configuration presents a p_f = 1.15x10^{-11}, indicating to be attending the high safety class limit by a large margin. Although connected to the same floater and under the same environmental conditions, the configurations present very different probabilities of failure. The main cause for this difference is the effect of the mean stress on the fatigue life. The static loads at the riser top for the free-hanging configuration are larger than those observed for the lazy-wave configuration. The quantification of these probabilities gives a support to the engineer to mitigate the risk when taking the decision whether to go for a design or another.

FORM analyses give results quite close to those predicted by MCSIS, resulting in a p_f = 1.451x10^{-5} for the Case 1 and a p_f = 1.25x10^{-11} for the Case 2, being a suitable approach for the reliability analyses of both cases studied, enabling the quantification of the importance factors. Figure 6 presents, for Cases 1 and 2, the importance factors of each random variable considering T_{oper}=30-yr and the target failure probability equal to 10^{-5} (High Safety Class), and Figure 7 shows the behavior of the importance factor values for each considered T_{oper}.

The importance factors have almost the same behavior for the free-hanging and lazy-wave configurations presenting a slightly difference for some variables. Palmgren-Miner's rule limit damage and the random variable associated to the S-N curve, together, contribute with around 75% of the total failure probability for both configurations, as shown in Figure 6. The random variables associated to friction coefficient, floater RAO and local analysis comes in the sequence, contributing with 7% for the failure probability, leaving all the other variables with a total contribution around or less than 3%.
The random variables representing the uncertainties in the static offset and in the armour wire tension distribution have more influence on the results obtained for the free-hanging configuration, while that representing the drag coefficient gains some prominence in the analysis of the lazy-wave configuration.
Figure 7 highlights the value stability and no changing of importance order along the operation time, but an exception is found for the friction coefficient (X_9) in Case 1, where mean tensions are higher. An increasing importance tendency can be observed as the operation time increases.

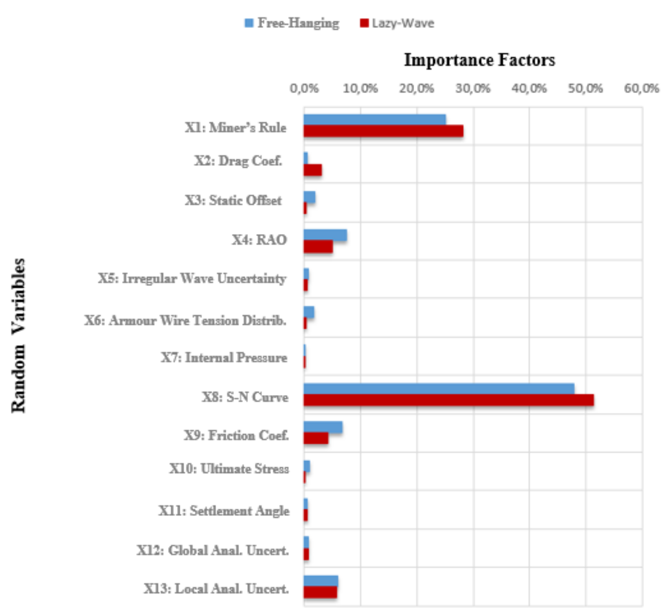

Figure 6 – Importance factors for the last year in operation

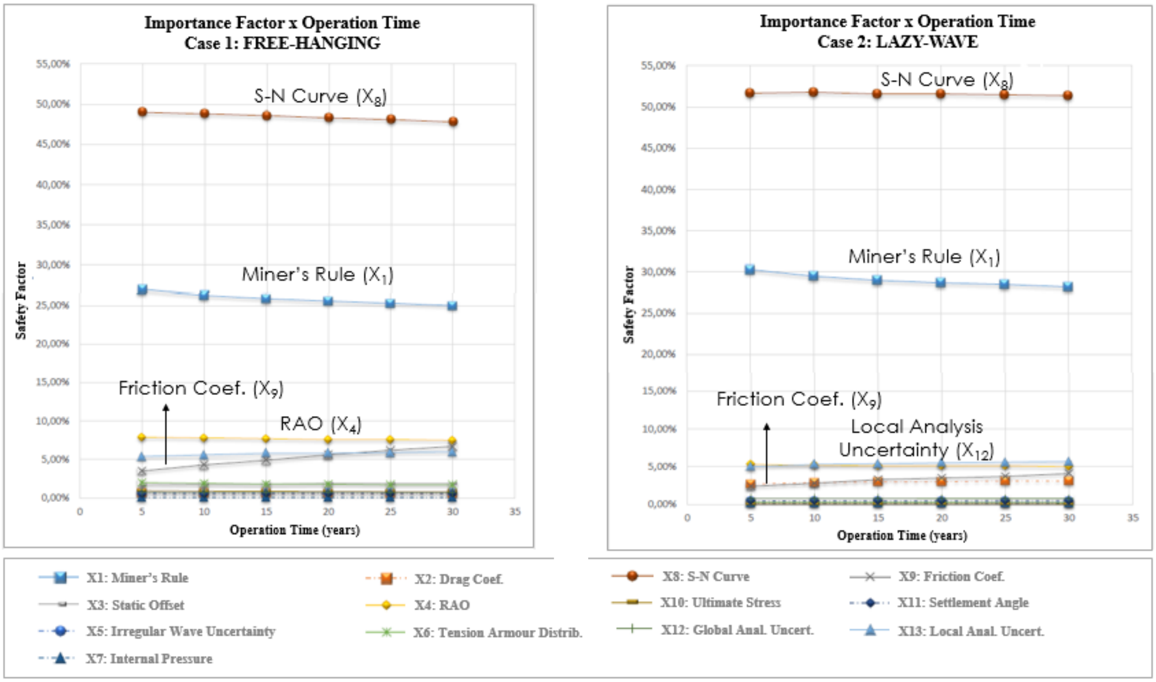

Figure 7 – Importance factor values' behavior for each T_{oper} (left: Free-Hanging; right: Lazy-Wave)

Figure 8 shows the safety factors as a function of the target probability of failure for both configurations, considering T_{oper}=30-yr and both reliability methods MCSIS and FORM. Table 2 presents the calibrated safety factors for the probability failure levels 10^{-3}, 10^{-4} and 10^{-5}. As it can be observed, the results are practically the same for both configurations, independently of the reliability method employed and configuration analyzed. It must be emphasized that these results reflect the uncertainties considered in the present study. The uncertainties representation employed here, mainly that associated to S-N curve, must be better understood before defining the reliability-based safety factors to be employed in the design practice.

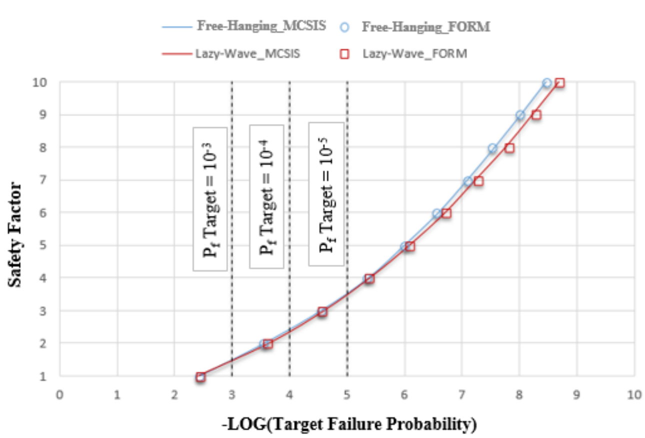

Figure 8 – Safety factor behavior for Cases 1 and 2

Table 2 – Study cases' safety factors

Configuration	Target Failure Probability	Calibrated SF (MCSIS / FORM)
Free-Hanging	10^{-3}	1.43 / 1.44
	10^{-4}	2.33 / 2.35
	10^{-5}	3.52 / 3.56
Lazy-Wave	10^{-3}	1.42 / 1.43
	10^{-4}	2.31 / 2.32
	10^{-5}	3.45 / 3.46

6. SENSITIVITY ANALYSIS RESULTS

Five sensitivity analysis were done in order to verify the impact on the reliability results, and each of them is presented below:

- Random variables amount reduction within the same analysis
- Assessment of new hypotheses assumed for Palmgren-Miners' rule damage limit (X_1) and S-N curve (X_8)
- Irregular wave global analysis with smaller time lengths (X_5)
- Importance and influence of the armour wires tension distribution (X_6)
- Inclusion of 3D load effect (X_{14}) random variable

For these analyses, only FORM was employed and T_{oper} is taken as 30-yr The results obtained in the previous section are designated here with the word ORIGINAL. Table 4 presents a summary of some of the sensitivity studies performed.

The time spent to complete all the analysis process' steps is very long, and the random variables that are dependent of normalized surface response functions contributes even more for the large time consuming. The random variables were reorganized by its decreasing importance factor values (as per results shown in Figure 6), giving a list starting from the most important. Twelve analysis, for each Case, are launched and each of them eliminates one random variable from the set, always considering the one with the lowest importance. Figure 9 shows the impact on the failure probability results.

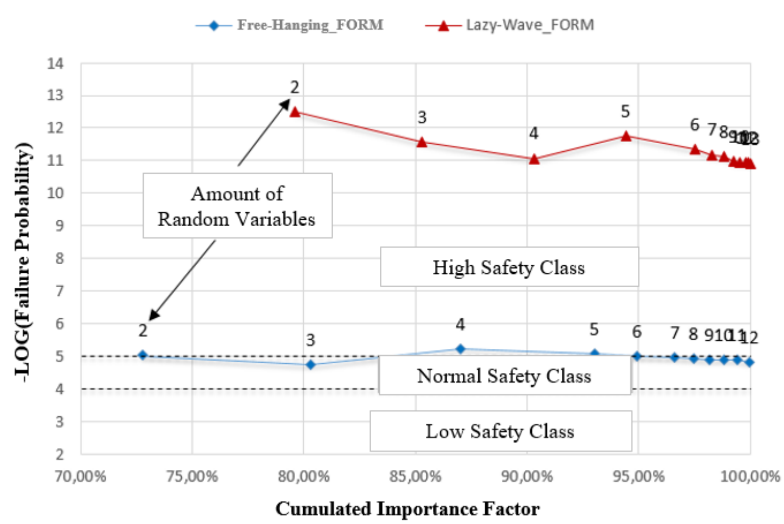

Figure 9 – Failure probability behavior due to different amount of random variables

When the problem handles with very small failure probabilities, the Lazy-wave configuration in this case, the results are very sensitive to any addition or removal of random variables. When looking to the Free-hanging configuration, its failure probability can be well estimated by considering just the two most important random variables: S-N curve and Palmgren-Miner's rule.

The random variables associated to the S-N curve and Miner's rule limit damage (X_8 and X_1), appears with the highest importance factors for both cases analyzed (see Figure 6). An increase/decrease of 20% upon its coefficient of variation is analyzed while keeping the other random variables with their original description shown in Table 1. The reliability results, shown in Table 4, confirms the expectation of being more sensitive to the change in the CoV of the random variable representing the variability of the S-N curve than in the Miner's rule fatigue damage limit, for both cases (free-hanging and lazy-wave configurations).

The use in the global riser analysis of shorter stochastic time domain simulations can speed up the whole fatigue analysis process, but, shorter simulations increase the variability in the fatigue damage results. The impact of this aspect on the reliability results is verified considering the statistical parameters for three simulation lengths: 500s, 1000s and 2000s. The fatigue damage uncertainty associated to each length (X_5) is taken from Figure 4. The results are presented in Table 3.

Table 3 – Calibrated safety factors under simulation time length variation

	Safety Factor (SF) Pf = 10^{-5}	
Simulation Length (s)	Case 1: Free-Hanging	Case 2: Lazy-Wave
500	3,80	3,58
1000	3,64	3,60
2000	3,57	3,52
3000	3,57	3,51
3800 (ORIGINAL)	3,56	3,46

Very short time-domain stochastic simulations (e.g. 500s long or less) have a high impact on the results. The calibrated SF is penalized due to the increase in the uncertainty of the variable. However, due to exponential behavior of the uncertainty associated to this aspect (see Figure 4), the optimal simulation length appears to be around 2000s, since the reliability analysis results are quite the same as those obtained with a simulation length of 3800s. Half of the time spent running global analysis could be saved once considering the optimal time.

Depending on the mounting process and the proximity of the riser's section analyzed relative to the end-fitting, the stress distribution (X_6) can amplify the dispersion of the stress distribution around the mean value by a factor of 2 or 3. In order to cover a wide range of statistical characterization possibilities, the mean and the coefficient of variation (CoV) of X_6 are varied from 0.8 to 1.2 and from 3.5% to 19% respectively, resulting the graphs shown in Figure 10.

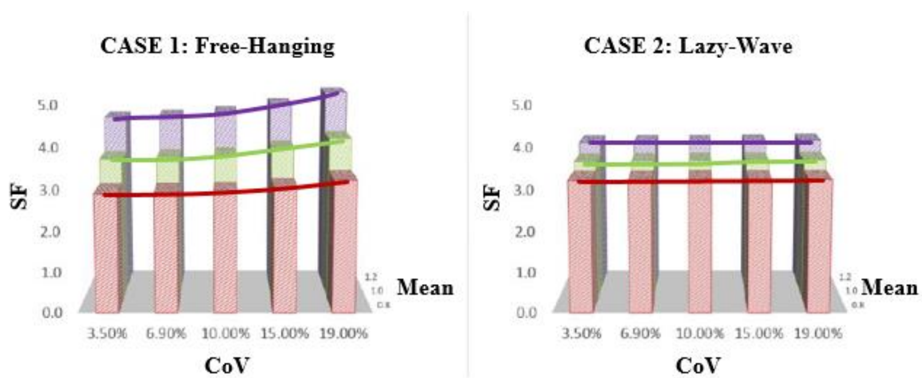

Figure 10 – SF x CoV for X_6 sensitivity analysis

The uncertainty associated to the armour wires tension distribution due to end fitting mounting imperfections has a larger impact on the free-hanging configuration than the lazy-wave one. This is also associated to the larger static loads found in the riser's top of the free-hanging configuration. Accurate assessment must be done in order to find the distance from the end fitting in which X_6 no longer influences reliability results.

The last sensitivity analysis includes to the original set the 3D load effect (X_{14}) random variable, considered in Refs. [2] and [14], which presents a high importance factor. It tries to represent the uncertainty related to the simplifications used to the model wind, currents and wave loads in the global analysis, once it is difficult to reproduce the multi-directional behavior of environmental conditions exactly as they occur in the nature [14]. No basin data of the study cases was available to characterize this random variable, and so, it has been considered as a Normal random variable having mean value, $\mu_{X_{14}} = 0.85$ and standard deviation of $\sigma_{X_{14}} = 0.1$ [2]. This uncertainty is defined directly related to the stress ranges and its normalized response surface function is given by [7]:

$$f(X_{14}) = (X_{14})^m \quad (13)$$

As per Table 4, the addition of X_{14} affects significantly the reliability analysis results for both cases analyzed. One must have in mind that the statistical representation used does not represent the real conditions of the study cases, but it indicates to be important its consideration into the set of random variables. This aspect claims for a better understanding on the statistical representation of this random variable for each field condition.

Table 4 – Study cases' safety factors

Sensitivity Analysis	Original results	X_1: Increase CoV. to 0.36	X_1: Decrease CoV. to 0.24	X_8: Increase CoV. by 20%	X_8: Decrease CoV. by20%	X_{14}: Added to the reliability analysis
P_f	1.45×10^{-5}* 2.39×10^{-11}**	2.75×10^{-5}* 3.45×10^{-10} **	6.99×10^{-6}* 1.19×10^{-12}**	4.39×10^{-5}* 4.69×10^{-10}**	3.76×10^{-6}* 1.02×10^{-13}**	2.18×10^{-4}* 8.47×10^{-9}**
Calibrated SF **P_f target = 10^{-5}**	3.56* 3.46**	4.00* 3.93**	3.21* 3.11**	4.38* 4.29**	2.94* 2.85**	4.92* 4.90**

*Free-Hanging configuration
**Lazy-Wave configuration

7. CONCLUSIONS

The fatigue of the armour wires at the bending stiffener region of two 6" flexible riser's configurations (free-hanging and lazy-wave) for the same floater under the same load conditions were analyzed using the structural reliability methodology. As expected, the lazy-wave configuration presents a higher safety margin (lower probability of failure) due to the lower level of mean stresses observed at the riser top.

The methodology employed also allows to obtain calibrated safety factors for a given target failure probability. These safety factors could be employed in similar applications without new reliability analysis. However, they reflect the level of statistical modelling employed in the analysis. As shown within sensitive analyses, some parameters may affect significantly on the calibrated safety factors, which resulted on values far below of 10 (which is a frequent SF used within the industry).
Since the reliability methodology is available, a better and rational understanding on the random variables to be considered, and their statistical description, can now be pursued, in order to establish reliability-based safety factors to be employed in design practice or to use a fatigue design methodology for flexible risers based entirely on structural reliability analysis, in a similar way as is available in Ref. [3].

8. REFERENCES

[1] Campello, G.C., 2014, "Design Methodology for End-Fitting's Anchoring System of Flexible Pipes and New Technology Proposal", D.Sc. Thesis, COPPE/UFRJ, Rio de Janeiro, RJ, Brazil (in Portuguese).
[2] Leira, B.J., Baarholm, G.S., Igland, R.T., Farnes, K.A., Percy, D., 2005, "Fatigue Safety Factors for Flexible Risers based on Case Specific Reliability Analysis", Proc. of the 24th International Conference on Offshore Mechanics and Arctic Engineering, OMAE67432, Halkidiki, Greece.
[3] Det Norske Veritas (DNV), 2005, "Recommended Practice DNV-RP-F204 – Riser Fatigue", Hovik, Norway.
[4] Det Norske Veritas (DNV), 2010, "Offshore Standard DNV-OS-F201 – Dynamic Risers", Hovik, Norway.
[5] Melchers, R.E., 2002, Structural Reliability Analysis and Prediction, 2 ed., John Wiley & Sons.
[6] Almar-Naess, A., 1999, Fatigue Handbook Offshore Steel Structures, Tapir, Trondheim, Norway.
[7] Filho, F.S.L., 2008, "Fatigue Analysis Methodology for Flexible Pipes Based on Structural Reliability", M.Sc. Dissertation, COPPE/UFRJ, Rio de Janeiro, RJ, Brazil (in Portuguese).
[8] Silva, V.R.M., 2015, "Fatigue Reliability Analysis for Armour Wires of Flexible Pipes", M.Sc. Dissertation, COPPE/UFRJ, Rio de Janeiro, RJ, Brazil (in Portuguese).
[9] Principia & IFP Energies Nouvelles, 2011, "DeeplinesTM 4.5 - Theory Manual", La Ciotat, France
[10] Feret, J., Bournazel, C.H., 1987, "Calculation of Stresses and Slip in Structural Layers of Unbounded Flexible Pipes", Journal of Offshore Mechanics and Artic Engineering (OMAE), 109, pp. 263-269.
[11] Feret, J., Leroy, J.M., Estrier, P., 1995, "Calculation of Stresses and Slips in Flexible Armour Layers with Layers Interaction", Journal of Offshore Mechanics and Artic Engineering (OMAE), V, pp. 469-474.
[12] Vignoles, M.A., 2002, "Deterministic and Random Analyses for Fatigue Life Evaluation of Flexible Risers' Metallic Armours", M.Sc. Dissertation, COPPE/UFRJ, Rio de Janeiro, RJ, Brazil (in Portuguese).

[13] American Society of Testing Materials (ASTM), 2011, "Standard Practices for Cycle Counting in Fatigue Analysis - E1049-85", West Conshohocken, USA.

[14] Filho, F.S.L., Lima, E.C.P., Sagrilo, L.V.S., Sousa, F.M.S., Lemos, C.A.D., 2012, "Safety Factors for Fatigue Analysis of Flexible Pipes Based on Structural Reliability", Proc. of the 31th International Conference on Offshore Mechanics and Arctic Engineering, OMAE83586, Rio de Janeiro, Brazil.