Negotiation of Extended Warranties for medical equipment with perfect and complete information

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1. INTRODUCTION

Hiring the Original Equipment Manufacturer (OEM) to execute maintenance services has been a common practice in hospitals and other healthcare institutions. As mentioned by Murthy & Jack [1], some reasons drive companies to outsourcing maintenance services rather than performing them in-house: 1) reduce costs, 2) improve service, 3) obtain expert skills, 4) improve processes and 5) improve focus on core activities. These advantages of outsourcing make the company look for other organizations that are able to provide maintenance, as commented by Jackson & Pascual [2].

Healthcare institutions employ a diverse range of equipment, under which some that are simple and others that use breakthrough-technology. The latter the interest of the present paper. They must reach high quality and reliability levels in order to ensure the availability of the service and the safety of patients, which may be achieved through very specific maintenance actions prescribed by the equipment manufacturer. As the OEM retains the specific resources and knowledge, not providing training to other maintenance providers, and not supplying sufficient spare parts to the market, it becomes difficult to hire other maintenance agents. Thus, the OEM emerges as the only feasible option for accomplishing the several pre-established maintenance procedures and standards to ensure equipment performance and service quality.

De Vivo *et. al.* [3] also emphasizes that some technologies used in medical equipment are so complex that performing in-house maintenance becomes unfeasible and hiring independent agents is difficult as they may not be familiar to the equipment's structure and may not have access to all necessary information to perform the maintenance service with the required procedures and quality standards defined by the equipment manufacturer.

2. OBJECTIVES OF THE WORK

Therefore, the present paper aims to tailor a mathematical model, which assists the hospital's decision maker (customer) to decide for the best maintenance policy and it is solved using a game-theoretical framework. The manufacturer and the customer (players) will interact in a Stackelberg game formulation according to which the former acts as a leader and the latter as a follower. Thus, the manufacturer decides first and then the customer makes his decision. This type of game fits well the situation modeled because the OEM acts as a monopolist and so has more bargain power than the customer. It has also been used by Murthy & Yeung [4], Murthy & Asgharizadeh [5], Asgharizadeh & Murthy [6] and Esmaeili. *et. al.* [7]. Despite that, the present work focuses on the maintenance of medical equipment, a field which is less explored according to Cruz & Rincón [8] and extends one of the models proposed by Murthy and Jack [1], by studying the specific field of medical equipment, with healthcare institutions dealing with the manufacturer to maintain an equipment to which it may be difficult to find other service providers rather the OEM and by considering two periods of EWs, each one with different failure probabilities, due to equipment degradation.

The customer's options are to purchase or not the equipment, and afterwards to hire an extended warranty (EW), for one or two periods, for a fixed price, or to fund all the losses by himself if a failure occurs and his equipment is not protected by an EW. For an equipment under EW, the losses caused by failures will be partially or completely refunded by the OEM, depending on the terms agreed by both parties.

Besides the two introductory sections, this paper unfolds as follows: in Section 3 the methodology is presented, highlighting the model proposed and its assumptions, in Section 4 the results are presented, with the optimal decision for the manufacturer and the customer and a numerical example to illustrate the model formulated, in Section 5 the results are discussed, some limitations and future contributions are pointed out.

3. DESCRIPTION OF METHOD

To deal with negotiation of EWs, game theory will be employed, following Murthy & Yeung [4], Murthy & Asgharizadeh [5], Asgharizadeh & Murthy [6], who studied maintenance service contracts using a Stackelberg game, Jackson & Pascual [2], who assessed maintenance service contracts using a Nash game, and Esmaeili et. al. [7], who evaluated maintenance service contracts and EWs using non-cooperative and semi-cooperative games. In the game formulated, the agents will be considered to be rational and to have perfect and complete information. They will interact in accordance with a Stackelberg game formulation and so it will be a static, non-cooperative, sequential game in which the OEM acts as a leader, is a monopolist in the maintenance service market and has more bargain power than the customer (healthcare institution), which will act as a follower.

3.1 Model Formulation

Besides the considerations regarding the game, which drives the interaction between the OEM and the hospital, now we point out other model's assumptions related to the medical equipment and the players' decision problems.

3.1.1 Equipment Failures and Repairs

The equipment considered in the present paper is an angiograph, a technology-intensive equipment which records patterns of pulse waves inside blood vessels through images. The hospital uses the referred equipment to generate revenue. It has a production cost of C_p and produces revenue of m_1 monetary units for the duration of the first EW and m_2 for the duration of the second EW, where $m_1 > m_2$. All failures that occur during the EWs will cause a loss of k monetary units. When covered by EW, the healthcare institution will be refunded by the amount $s_{w1}k$, or $s_{w2}k$ by the manufacturer, respectively, to partially or completely cover the losses caused by failure occurrences, where s_{w1} and s_{w2} represent the level of protection – percentage of the loss caused by a failure (k) which must be refunded to the customer – offered by the EW and may assume any value between 0 and 1, where 0 would represent no coverage and 1 total coverage of losses.

All units produced are homogeneous w.r.t. their reliability characteristics. Failures occur with probabilities $(1 - \pi_{w_1})$ and $(1 - \pi_{w_2})$ during the length of the first and second EW, respectively. We consider a different probability of failure in each stage to consider in our model that the equipment degrades over time. Due to equipment degradation, the probability of failure is higher during the second EW length, thus $\pi_{w_1} > \pi_{w_2}$. After the occurrence of a failure, the equipment is subject to perfect repair.

3.1.2 Customer's Decision Problem

The customer also intends to maximize his expected profit and faces the problem of deciding whether to buy the equipment or not, and of hiring EW or not, renewing the EW or not, and, when not covered by any legal agreement, of funding all the maintenance costs after a failure occurs with his own resources. We assume that each EW has the same length, and that the second EW begins immediately after the first expires. Thus, the customer's decision variable x may assume four values for each possible decision: 0) for not buying the equipment, 1) for buying the equipment and not signing an EW with the OEM, 2) for buying the equipment and hiring the EW for one period, therefore funding all losses caused by failure occurrences after the EW expires and 3) for buying the equipment and hiring the EW for two periods.

The manufacturer's and customer's expected profits will describe the objective functions of the problem formulated. The problem is solved by backwards-induction, Osborne & Rubinstein [9]. Thus, although the manufacturer makes his decision formerly, the optimization problem of the customer must be resolved first. Afterwards the OEM's problem may be solved, taking into account the optimal values found in the customer's problem, and the optimal choices of both will then be determined.

The customer's expected profit in each situation is given by (1):

$$L_C = \begin{cases} 0; & if\ x=0, \\ m_1+m_2-P_p-[(1-\pi_{w1})+(1-\pi_{w2})]k; & if\ x=1, \\ m_1+m_2-P_p-P_{w1}-[(1-\pi_{w1})(1-s_{w1})+(1-\pi_{w2})]k; & if\ x=2, \\ m_1+m_2-P_p-P_{w1}-P_{w2}-[(1-\pi_{w1})(1-s_{w1})+(1-\pi_{w2})(1-s_{w2})]k; & if\ x=3. \end{cases} \quad (1)$$

Thus, the customer's maximization problem is as follows:

$$max\ L_C(x;y) \quad (2)$$

$$s.t. P_p \leq m_1 + m_2 - [(1-\pi_{w1}) + (1-\pi_{w2})]k \quad (3)$$

$$P_{w1} \leq (1-\pi_{w1})s_{w1}k \quad (4)$$

$$P_{w2} \leq (1-\pi_{w2})s_{w2}k \quad (5)$$

$$P_p, P_{w1}, P_{w2} > 0 \quad (6)$$

$$0 \leq s_{w1} \leq 1 \quad (7)$$

$$0 \leq s_{w2} \leq 1 \quad (8)$$

The constraint (3) shows that the customer will buy the equipment only if the price he pays for it is lower than the expected profit he earns by using it. In other words, if $L_C\ (1;\ y)\ >\ L_C\ (0;\ y)$ and will be indifferent between the two options if $L_C\ (1;\ y)\ =\ L_C\ (0;\ y)$. The constraint (4) models the decision of whether or not to hire the EW in period 1. The customer will hire it if $L_C\ (2;\ y)\ >\ L_C\ (1;\ y)$ and will be indifferent if $L_C\ (2;\ y)\ =\ L_C\ (1;\ y)$. The constraint (5) considers the decision of whether or not to hire the EW in period 2. The customer will hire it if $L_C\ (3;\ y)\ >\ L_C\ (2;\ y)$ and will be indifferent if $L_C\ (3;\ y)\ =\ L_C\ (2;\ y)$. Thus, the customer will be indifferent about hiring an EW for period 1 and period 2 when $P_{w1}(s_{w1})\ =\ (1 - \pi_{w1})\ s_{w1}k$ and $P_{w2}(s_{w2})\ =\ (1 - \pi_{w2})\ s_{w2}k$ respectively, note that both EW prices are functions of their respective level of protection s_1 and s_2, though, due to a notation simplification, these arguments will be omitted from here on. The constraints (6), (7) and (8) restrict the values that may be assumed by each decision variable.

3.1.3 *Manufacturer's Decision Problem*

The manufacturer faces the problem of determining the equipment sell price, P_p, the EWs prices, which are functions of their levels of protection s_{w1} and s_{w2}, in order to maximize his profit. The EWs state that for fixed prices P_{w1} and P_{w2} the customer should be refunded of an amount $s_{w1}k$ or $s_{w2}k$ from the manufacturer, proportional to the level of protection previously agreed. All manufacturer's decision variables will, when convenient, be condensed in variable $y \equiv (P_p, s_{w1}, s_{w2}, P_{w1}, P_{w2})$.

The manufacturer's expected profit in each situation is given by (9):

$$L_M = \begin{cases} 0; & if\ x = 0, \\ P_p - C_p; & if\ x = 1, \\ P_p - C_p + P_{w1} - (1 - \pi_{w1})s_{w1}k; & if\ x = 2, \\ P_p - C_p + P_{w1} + P_{w2} - [(1 - \pi_{w1})s_{w1} + (1 - \pi_{w2})s_{w2}]k; & if\ x = 3. \end{cases} \quad (9)$$

Thus, the manufacturer's maximization problem is as follows:

$$max\ L_M(y; x^*) \quad (10)$$

$$s.t. P_p > C_p \quad (11)$$

$$0 \leq s_{w1} \leq 1 \quad (12)$$

$$0 \leq s_{w2} \leq 1 \quad (13)$$

$$x^* = argmax\ L_C(x; y) \quad (14)$$

The constraint (11) shows that it will only be economical for the manufacturer to sell the equipment if its price is higher than its production cost. Constraints (12) and (13) limit the level of protection of the EW in periods 1 and 2 to values between 0 and 1. The constraint (14) signals that the OEM takes the decision of the customer into account while determining the optimal values of his decision variables. Thus, the manufacturer should select the optimal set of his decision variables y^* so that the customer is willing to buy the equipment and hire the EW for both periods, while the manufacturer maximizes his own expected profit. Thus, it must satisfy $L_C\ (3;\ y) = L_C\ (2;\ y) = L_C\ (1;\ y)$ and also maximize $L_M = (y;\ x^*)$.

4. RESULTS

4.1 *Manufacturer's Optimal Strategy*

After solving the manufacturer's optimization problem and deriving the optimal values of his decision variables, we have found that the optimal strategy for the manufacturer is to sell the equipment to the customer, but he would be indifferent to provide him maintenance services due to offering an EW for one or two periods. This is so because in all those three situations his expected profit would be maximized, and thus, assume the same value. In figure 1 the customer's decision regions are presented, the blue line represents the customer's indifference points, which are sought by the manufacturer as they maximize his profit.

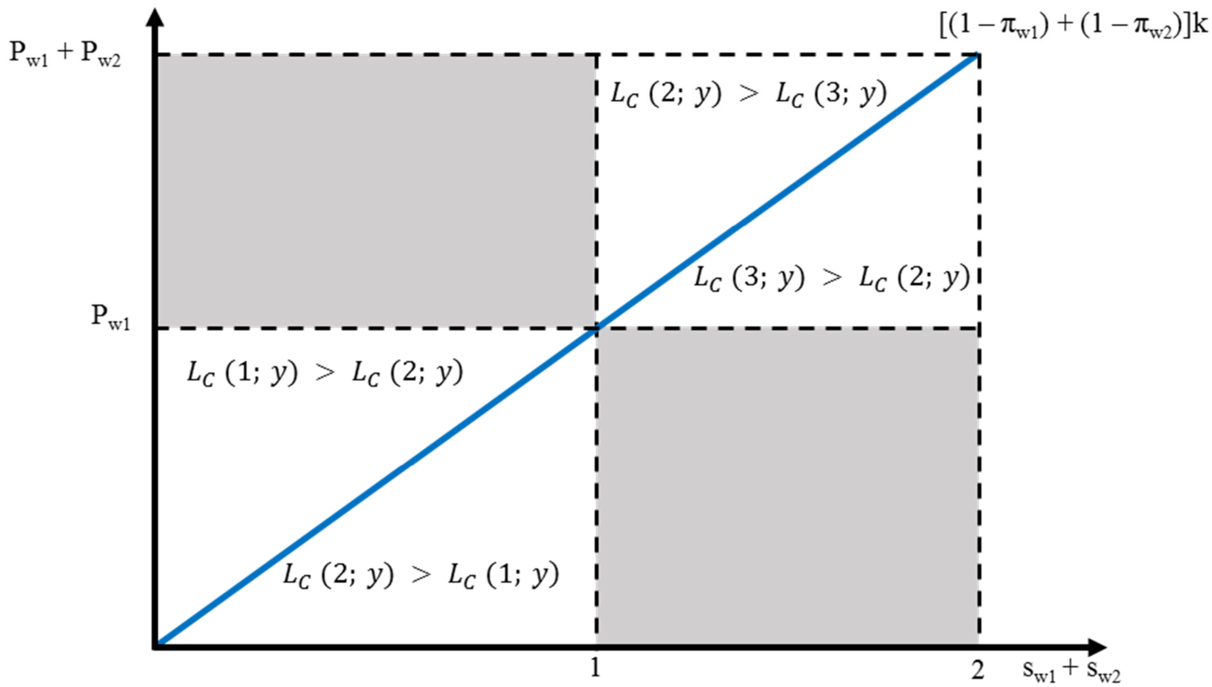

Figure 1. Decision regions for periods 1 and 2

As it can be observed in equation (15), the manufacturer's optimal expected profit does not depend on the values of the levels of protection s_{w1} and s_{w2}, thus they may assume any value between [0;1]. However, as the EWs prices depend on those values, they will interfere on the determination of P_{w1} and P_{w2}, respectively. This result is equivalent to the result found by Murthy and Jack [1]. Thus, the manufacturer's optimal strategy may be summarized as follows: first he determines $P_p = m_1 + m_2 - (2 - \pi_{w1} - \pi_{w2})k$, then s_{w1} and $s_{w2} \in [0,1]$ in order to determine $P_{w1} = (1 - \pi_{w1})s_{w1}k$ and $P_{w2} = (1 - \pi_{w2})s_{w2}k$. Thus, his optimal expected profit would be given by equation (15), which shows that the manufacturer's expected profit will be given by the difference between the EW's price and the equipment's production cost (C_p):

$$L_M(y^*;1) = L_M(y^*;2) = L_M(y^*;3) = P_p - C_p = m_1 + m_2 - (2 - \pi_w - \pi_c)k - C_p \tag{15}$$

Thus, the manufacturer's expected profit will be an increasing function of P_p in accordance with the result found by Murthy and Jack [1].

4.2 *Customer's Optimal Strategy*

As we have used a Stackelberg game formulation with complete and perfect information, the manufacturer is able to predict the customer's decisions as well as his maximum willingness-to-pay for each of the options offered. Thus, the OEM is able to extract all the consumer surplus and leaves him with an expected economic profit of zero. Thus, the customer's optimal strategy is to purchase the equipment, but then he will be indifferent between hiring the EW for one or two periods. This situation will maximize the manufacturer's expected profit and may be observed in equation (16), which shows that the expected economic profit of the customer to all his possible decisions will be zero:

$$L_C(1; y^*) = L_C(2; y^*) = L_C(3; y^*) = 0 \qquad (16)$$

The concept of economic profit considers besides the explicit expenses, also the implicit ones, i.e. it takes into account the opportunity costs, defined as the combined costs of alternatives that must be forgone when a given alternative is selected Varian [10]. Once again, this result is equivalent to what was found by Murthy and Jack [1].

4.3 Numerical Example

In order to illustrate the presented model, we apply it to real data provided by a public hospital. The input parameters considered are displayed in table 1.

TABLE 1: Input parameters

Parameter	Value
m_1	$ 1200000.00
m_2	$ 1000000.00
k	$ 400000.00
π_{w1}	0.9
π_{w2}	0.8
s_1	0.9
s_2	0.8
C_p	200000.00

The first decision variable to be calculated is the equipment's sale price (P_p), using equation (3), its value is $ 2080000.00. It represents the maximum price the customer is willing to pay for the equipment and the manufacturer is only capable of charging exactly this value because of the structure of a game of perfect and complete information. Afterwards the warranty prices for periods 1 (P_{w1}) and 2 (P_{w2}) may be determined using equations (4) and (5); their values are $ 36000.00 and 64000.00 respectively. They represent the maximum price the customer is willing to pay for each EW and the manufacturer is only capable of charging exactly these values because of the structure of a game of perfect and complete information. Then, we need to determine the customer's expected profit (L_C) and the manufacturer's expected profit (L_M) using equations (1) and (9); their values were $ 0 and $ 1880000.00. It is important to emphasize that we consider the economic profit of each player, and thus all the opportunity costs for each of them. It means that an economic profit of zero makes it still attractable to the customer to buy and use the equipment. A summary of the results may be seen in table 2 below.

TABLE 2: Output – Decision Variables

Decision variable	Value ($)
P_P	2080000.00

P_{w1}	36000.00
P_{w2}	64000.00
L_C	0
L_M	1880000.00

In table 3 it is possible to see the changes on the decision variables due to variations on the customer's loss due to failures parameter (k). It is possible to see that for lower values of k the manufacturer's expected profit grows. Therefore, for higher levels of k the manufacturer's expected profit reduces. It may also be seen that as the value of k grows, the equipment price sinks, while the warranty prices for periods 1 and 2 maintain a growing trend. The value of the customer's profit is always zero, as mentioned in the previous section, due to the game characteristics of complete and perfect information and due to the consideration of economic profit.

TABLE 3: Output changes due to variations on the customer losses parameter (k)

Customer losses	k	300.000	350.000	400.000	450.000	500.000
Equipment price	P_P	2.110.000	2.095.000	2.080.000	2.065.000	2.050.000
EW price (period 1)	P_{w1}	27.000	31.500	36.000	40.500	45.000
EW price (period 2)	P_{w2}	48.000	56.000	64.000	72.000	80.000
Customer's profit	L_c	0	0	0	0	0
Manufacturer's profit	L_M	1.910.000	1.895.000	1.880.000	1.865.000	1.850.000

In order to provide an alternative visualization of the impact of variations on the customer's losses due to failures parameter (k), some charts are presented below. In figure 2, it may be seen that the equipment price sinks from \$ 2110000.00 to \$ 2050000.00 as k grows, a strictly decreasing behavior, it is so because higher values of k increase the term $[(1-\pi_{w1})+(1-\pi_{w2})]k$, which is negative (3), the equation used to determine P_P. In figure 3, the behavior of the extended warranty for period 1 price is provided, it rises from \$ 27000.00 to \$ 45000.00 as k increases, while in figure 4 the behavior of the extended warranty price for period 2 is provided, it rises from \$ 4800.00 to \$ 80000.00 as k increases, in both cases a strictly increasing behavior is observed, it is so because higher values of k cause higher values of equations (4) and (5), which are used to determine P_{w1} and P_{w2} respectively . In figure 5, it may be observed that the manufacturer's profit decreases from \$ 1910000.00 to \$ 1850000.00 as k increases, showing a strictly decreasing behavior, it is so because higher values of k increase the term $[(1-\pi_{w1})s_{w1}+(1-\pi_{w2})s_{w2}]k$, which is negative in (9), the equation used to determine the manufacturer's profit. All variables present a linear behavior as linear model has been proposed in the present paper.

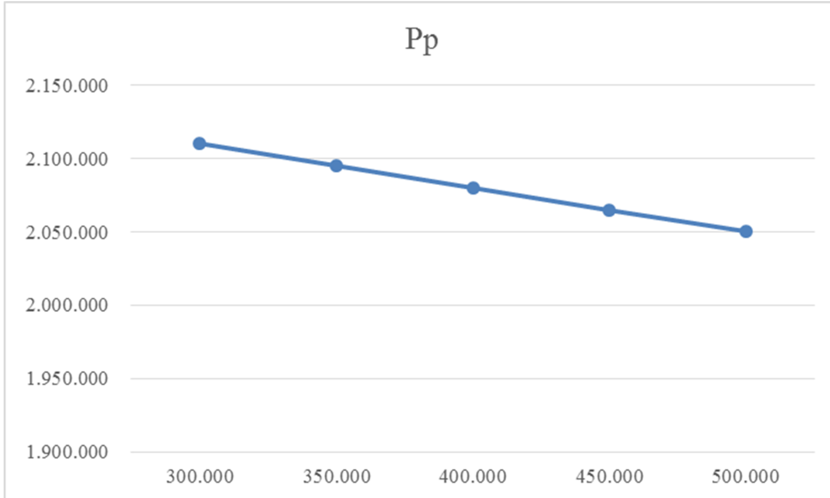

Figure 2. Changes on the equipment price (P_P) due to variations on k

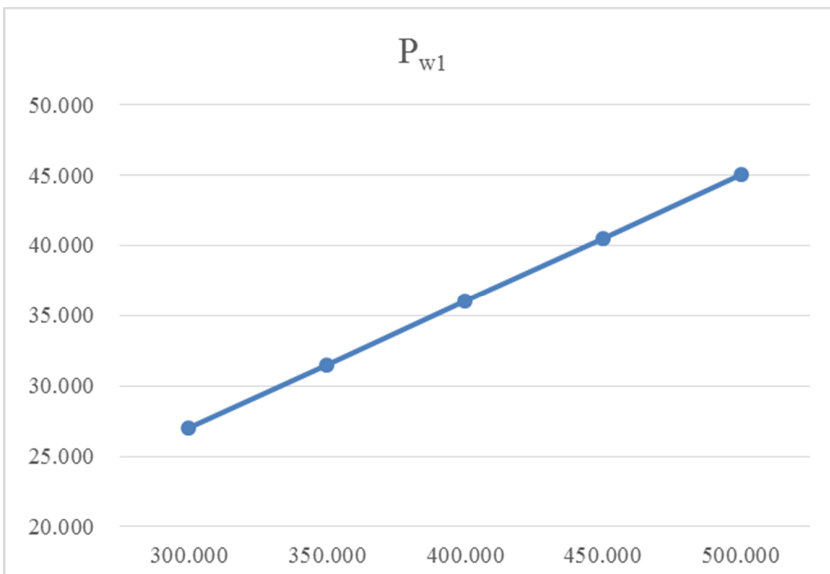

Figure 3. Changes on the extended warranty for period 1 price (P_{w1}) due to variations on k

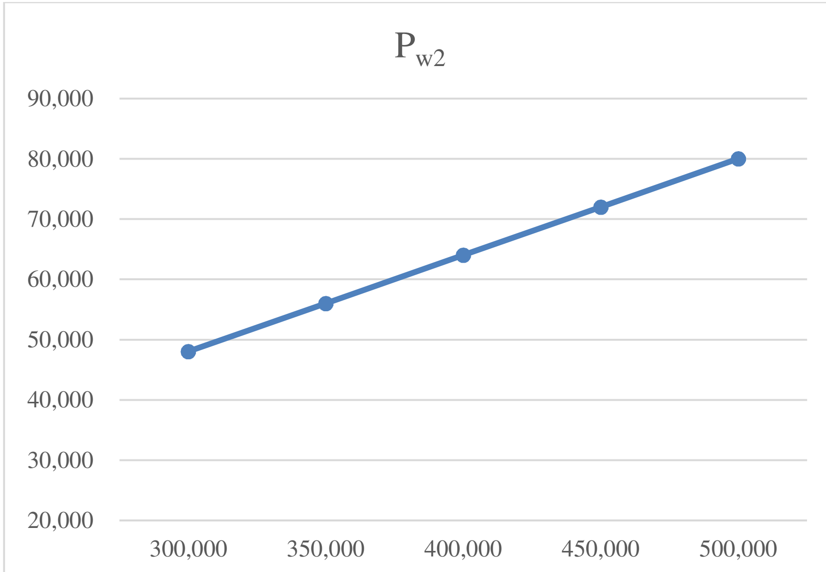

Figure 4. Changes on the extended warranty for period 1 price (P_{w2}) due to variations on k

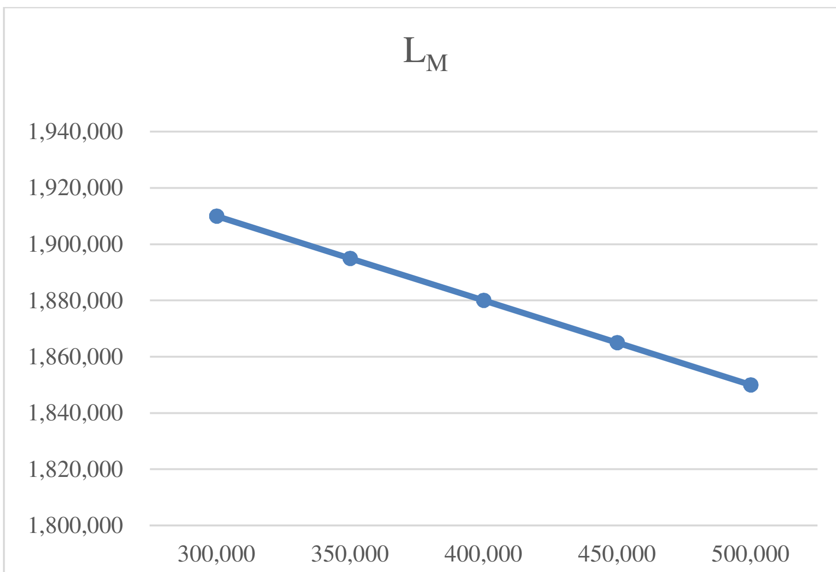

Figure 5. Changes on the manufacturer's profit (L_M) due to variations on k

5. FINAL COMMENTS

The model developed in the present paper is an extension of one of the models presented by Murthy and Jack [1], and intended to illustrate the situation in which a healthcare institution needs to acquire and maintain a technology-intensive equipment in order to service its patients. The extensions were: to study the specific field of medical equipment, with healthcare institutions dealing with the manufacturer to maintain an equipment to which it may be difficult to find other service providers rather the OEM and to consider two periods of EWs, each one with different failure probabilities, due to equipment degradation. Although the model has many simplifying assumptions, such as considering perfect and complete information, considering π_{w1} and π_{w2} constants and a period's revenue and loss m and k, it is an important starting point for more complex and realistic models in the future.

The model can be extended in several ways. Some of them are: (i) using a principal-agent game formulation to consider information asymmetry; (ii) considering manufacturer or customer to be risk averse; (iii) considering two-dimensional warranties to take other parameters such as equipment reliability or availability into account while negotiating; (iv) considering game of imperfect information and (v) considering multiple customers.

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