Extended Warranty of Medical Equipment Subject to Imperfect Repair: An Approach based on Generalized Renewal Process and Stackelberg Game

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1. INTRODUCTION

Medical equipment plays an important role in modern healthcare institutions because they present the following purposes: diagnosis, study, disease prevention, monitoring and patient treatment. During the last decades, as technology has advanced, the maintenance of such equipment has become too complex to be done in-house. Therefore, this activity has been commonly assigned to the Original Equipment Manufacturer (OEM).

In this context, according to [1] both the OEM and the hospital managers must ensure the following information is considered during the acquisition process of high-technology devices: operation manual, software updates, spare parts available throughout equipment life, exchange policy, service reports provided by manufacturer's service representative, and the warranty policy. The importance of studying more deeply the warranty issue for healthcare equipment is reinforced by the role maintenance management plays in guaranteeing quality of healthcare services. In diverse situations, human lives depend on the correct operation of medical devices. Its economic impact is also of importance; in the USA, for instance, more than 2 billion dollars are spent annually for maintenance of medical devices ([1]).

Being of interest to OEM and hospital managers, a warranty policy defines responsibilities for both parties. Indeed, the OEM is responsible to repair the devices' eventual failures related to problems of equipment design, manufacturing and/or quality. The customers in turn should make proper use of equipment; in other words, they must comply to the specifications defined by the OEM ([2]).

Associated with this practice, a new trend has been intensified by manufacturers, selling an additional, optional coverage, which begins after the expiration of the base warranty, called Extended Warranty (EW) ([3]). Thus, the customer decides whether to pay an extra value at the purchase epoch ([4]), whereas the OEM will correctively maintain equipment for a given period even after the ordinary warranty expires.

It is important to emphasize that EW ends up creating a conflict of interests between the owner of the equipment and the OEM. Specifically, the customer needs a high availability of its equipment at a reasonable servicing cost, whereas the manufacturer intends to maximize their profit with the addition of post-selling service. Consequently, EW is crucial for both parties, once that actions of one interferes in the results of the other. Due to this conflict of interests, Game Theory provides an appropriate approach to solve this problem ([5]).

Among different games that can be used to model the interaction between agents, the leader-follower Stackelberg Game (SG) is a good option for modeling hospital equipment maintenance service contracts. In fact, the manufacturer is commonly the only party able to perform maintenance, since it has the technical knowledge, expertise, technology and spare parts for the repair execution ([6]). The hospital, in turn, has the need of having its medical device available in suitable condition to provide a good service. From these two perspectives, an uneven power relationship is noted, which can be modeled via SG. The leader role is assigned to the OEM, which determines the terms of the EW; the health institution acts as the follower, responding to actions taken by the OEM.

Quantitative studies about warranty, maintenance outsourcing and maintenance contracts are present in [7], [8], [9] and [10]. However, such studies have restrictive and unrealistic assumptions with respect to the state of the system after a repair intervention. In fact, those papers considered that the system returns to either an "as good as new" condition (perfect repair) or an "as bad as old" condition (minimal repair); these two situations are modeled respectively according to a Renewal Process (RP) and a Non-Homogeneous Poisson Process (NHPP). [11] describes RP and NHPP in details.

The use of these assumptions about the system state after the maintenance action may result in inadequate managerial decisions, which can result in significant losses in company profits because of incorrect definition of warranty policies. In practical terms, maintenance actions typically return the equipment to an intermediate condition between the perfect and minimal repairs, which is called imperfect repair ([12]; [13]). [12] proposed two methods to tackle imperfect repairs: Kijima type I e Kijima type II, which gave rise to the Generalized Renewal Process (GRP) and introduced the concept of "virtual age". Furthermore, these situations generalize RP and NHPP; other approaches may be seen in [14].

The issue of the acquisition of EW of complex healthcare equipment using Stackelberg Game was detailed by [15], those authors considered the interaction between healthcare institutions and the OEM, modeling priority queues and Weibull distributed times until failures; those failures, however, are minimally repaired. The present paper intends to expand the problem using the GRP to model imperfect repair – a more realistic and general assumption. This approach will be applied to an interaction with multiple customers. By employing this methodology, we present a more robust model, which aims to be more adequate to situations encountered in practice, while still modeling situations described by the aforementioned work.

The proposed model will be characterized along these lines: First, we considered the OEM offers to the hospital managers two maintenance options for the period after the ordinary warranty expires: (i) an extended warranty or (ii) on-call service. Extended warranties states that for a fixed price P, the OEM should repair all failures without any additional cost for the hospital over the period of the contract; if a failed unit does not get repaired before a set time τ, a penalty, which increases over time, will be incurred. The on-call service, in turn, failures will be repaired at a fixed cost C_s each, and there is no penalty involved or a legal agreement between the two parties.

The remainder of this paper unfolds as follows. In Section **Erro! Fonte de referência não encontrada.** , the theoretical background is provided, containing the adaption of the Stackelberg Game for the context of maintenance contracts, as well as the characteristics of GRP, emphasizing the method proposed by [16]. Section **Erro! Fonte de referência não encontrada.** presents the proposed model, the players' optimal strategies and the equilibrium of the game. In Section **Erro! Fonte de referência não encontrada.**, a numerical example is presented, using real data from an angiograph, which is a device used for blood vessels mapping and making diagnosis of organ diseases. Finally, Section **Erro! Fonte de referência não encontrada.** brings some concluding remarks.

2. THEORETICAL BACKGROUND

2.1 Stackelberg Game

SG is a non-cooperative sequential game developed by Heinrich von Stackelberg in a leader-follower interaction. It was originally proposed to evaluate the equilibrium of a duopoly, where competing companies decide the optimal quantity to be produced ([17]).

At the best of authors' knowledge, [18] were the first authors to introduce the SG as an important tool to model maintenance service contracts. [19] and [20] incorporated multiple customers and service channels. [21] considered a three-level service contract between a manufacturer, a customer and an independent third agent.

To adapt the SG formulation for healthcare environment first we need to define the power structure between parties. Generally, in the context of complex medical equipment, the OEM has a well-trained staff, spare parts and dominates the equipment technology. Thus, OEM behaves as a leader, acts first and is the only maintenance service provider. Hospitals need to guarantee minimum levels of availability for their equipment. However, they do not have expertise in the maintenance of complex equipment. Therefore, hospitals can be considered as followers, once they react to the OEM's action.

The SG's solution can be achieved via backward induction, i.e., first the decision problem of the follower is solved, and then that result is used on the leader's profit function. Next, we can find the decisions that maximize each agent's objective function.

2.2 Generalized Renewal Process

RP and NHPP may be adopted to model perfect and minimal repair situations. Such methods have simplifying assumptions that may be unreal in many practical situations, like the healthcare technology intensive environment ([22]). To overcome on the limitations of RP and NHPP, [12] developed a probabilistic virtual age based model, known as the Generalized Renewal Process (GRP) that deals with all classes of corrective maintenance actions. According to this model, q (rejuvenation parameter) may generally assume values between 0 and 1:

- $q = 0$ represents a perfect repair (as good as new);
- $q = 1$ corresponds to a minimal repair (as bad as old);
- $0 < q < 1$ indicates imperfect repair (better than old, worse than new).

Cases where $q < 0$ and $q > 1$ are also possible corresponding to *worse than old* and *better than new* conditions, respectively. Generally, GRP may be classified into two types (Kijima Type I and II), according to the method used to calculate the virtual age. These types can be seen in details in [23], and other virtual age-based representations could be found in [24], [25] and [26].

This paper uses Kijima type I, which states that maintenance actions only act the very last failure and compensate the damage accumulated between the $(i-1)^{\text{th}}$ repair and the i^{th} failure; thus, the virtual age v_i proportionally increases over time. The calculation of virtual age, Cumulative Density Function (CDF) conditioned on virtual age can be seen in [22]:

For our analysis, we consider time until the first failure follows a Weibull distribution because of its flexibility and ability to fit various degradation stages. When there are reasonably sufficient failure data available, Maximum Likelihood Estimators (MLE) can be used to find GRP parameters α, β, and q. To this end, the procedure described in [16] can be followed, allowing for the estimation of GRP parameters for a device of interest.

2.3 Queueing Theory

Queues are part of everyday life, as we can see it in many situations such as markets, banks, gas stations and multiple other examples. They are formed when demand is higher than the system's service capacity for a certain time, causing in long waits for the customer or product on hold. Queuing Theory is the study of stochastic processes of demand and service, the relation between the demand on a given system and the delays faced by the systems' users.

In order to perform queuing system analysis, some information is needed, such as the users' arrival process, the queue discipline and the service process ([27]). In the proposed model, we use the process detailed in previous section, GRP to describe the arrival process for each user. The queue discipline follows a FCFS – first come first served – logic, by which users are served by order of arrival, i.e., devices start being repaired following the same order they failed. Times to repair follow an exponential distribution. Using the conventional notation, the queue can be described as G/M/1.

Analytical solutions are usual in cases of queuing models based only in exponential distributions, however it is more difficult when we consider more complex methods, such as Weibull-distributed times until failures, as well as the addition of game formulation into the problem ([15]). Therefore, a Discrete Event Simulation (DES) algorithm is adopted to obtain queue measures.

3. PROPOSED MODEL

This paper aims to determine the best strategies for the problem of membership or not to an EW applied on a medical context. This interaction between the OEM and each customer will employ a Stackelberg formulation and the game's equilibrium will be reached by finding the reservation prices of the customers (maximum prices that customers accept to pay for a good or service). This decision problem extends the model developed by [15] by considering imperfect repairs. Additionally, the problem is stochastic due to the uncertainty inherent to the presence of random variables in the model. Finally, decision makers (healthcare institutions and OEM) each have their own objective function that will define their respective payoffs. Both OEM and healthcare institutions are aware about their alternatives, acting rationally, choosing strategies that maximize their respective payoffs ([28]).

3.1 Notation

3.2 Problem Description

The OEM sells a technology-intensive medical device to multiple customers (hospitals) at the cost of C_b per unit. Each device, when in operational state, generates a revenue of R monetary units per time unit. The availability in this case is crucial to profit generation.

In the first period, the OEM provides a base warranty along with the purchase of a device, i.e., it is responsible for all repairs required with no charge to the client. If a failed unit is not returned to operational state within a period τ after a failure occurs, the OEM is charged a penalty proportional to the overtime in repairing the equipment, T_{ov}, which is the period between τ and when the equipment returns to operation. Therefore, the penalty is $\theta_1(Y_{ij} - \tau)$ when $Y_{ij} > \tau$; zero, otherwise. θ_1 is the penalty per unit time during overtime; Y_{ij} is the time between the occurrence of the i^{th} failure of equipment j and the completion of its respective repair. This penalty exists because medical equipment is vital for patients' treatment and for the hospital's profit. Then, unavailability affects their payoff and reputation.

At the time of purchase, each hospital may choose a repair service mode for after the expiration of the base warranty: i) Extended Warranty (EW) or ii) On-Call Service. These options are described as follows:

i) A_1: Extended Warranty – begins after the base warranty expires and has duration W_2. The customer pays a fixed price P and the OEM repairs all failed units over W_2 at no additional cost. If a failed unit is not returned to operational state within a period τ after a failure occurs, the OEM is charged a penalty. Analogously to the base warranty, the penalty is $\theta_2(Y_{ij} - \tau)$, when $Y_{ij} > \tau$;

ii) A_2: On-call Service – in this modality, the OEM executes each repair at a cost of C_s per intervention; no penalty is incurred in this modality.

In the case of option A_0, the hospital chooses not to buy the equipment.

3.3 Hospital's decision problem

The healthcare institutions must make two decisions at the same time. Indeed, considering the options presented by the OEM, the hospital must decide whether to opt for the EW (A_1), on-call service (A_2) or not to purchase the equipment (A_0), where A_0 will occur if their expected utility becomes inferior than zero over the period W (participation restriction).

Each strategy has consequences to the payoffs. Thus, the choice is strongly affected by equipment reliability, pricing structure and the hospital's degree of risk aversion (δ). The customer's risk is modeled according to a utility function U. The utility function is important because it indicates how the customer chooses among distinct options; thus, the preferred options are represented by higher utilities. The utility function considered in this model has been used in [29] showed in eq. (1111):

$$U(\Pi) = \left(\frac{1-e^{-\delta\Pi}}{\delta}\right), \text{ where } \Pi \text{ represents the associated wealth.} \tag{1}$$

The degree of risk aversion increases as δ grows. All customers are assumed homogeneous with respect to their risk aversion. All equipment units are identical regarding their reliability.

Each customer's return, $\Pi_H(A_k)$ depends on the option A_k chosen by the customers, thus, the hospitals profit can be seen in eqs. (2), (3) and (4).

$$\Pi_{HA_0} = 0 \tag{2}$$

$$\Pi_{HA_1} = RT_{op} + \theta_1 T_{ov_1} + \theta_2 T_{ov_2} - P - C_b \tag{3}$$

$$\Pi_{HA_2} = RT_{op} + \theta_1 T_{ov_1} - C_b - \sum_{j=1}^{M} N(W_{2j})C_s \tag{4}$$

Where: $T_{op} = \left(\sum_{i=1}^{N(W_j)} X_{ij} + \widetilde{X}_j\right)$; $T_{ov_1} = \left[\sum_{i=1}^{N(W_{1j})} \max\{0, (Y_{ij} - \tau)\}\right]$; $T_{ov_2} = \left[\sum_{i=1}^{N(W_{2j})} \max\{0, (Y_{ij} - \tau)\}\right]$; $N_{f2} = \sum_{j=1}^{M} N(W_{2j})$.

For customer j, $(1 \leq j \leq M)$, let the number of failures be N_j. X_{ij} $(0 \leq i \leq N_j)$ denotes the time between the i_{th} repair and the $(i+1)_{th}$ failure for the j_{th} equipment ; $\tilde{X}$ indicates the time that the equipment is operational after the warranty period; $N(W_{1j})$ is the number of failures occurred over $[0, W_1]$ for the j_{th} equipment; $N(W_{2j})$ is the number of failures occurred over $[W_1, W_2]$ for the j_{th} equipment. y_{ij} $(0 \leq i \leq N_j)$ is the total time to finish repairing the j_{th} equipment since the occurrence of the i^{th} failure, i.e., y_{ij} includes the waiting time in queue.

3.4 OEM's decision problem

The OEM is considered risk neutral and its expected profit is related to the customer's optimal choice. Consequently, the OEM's payoff can be denoted as $\Pi_{OEM}(P, C_s, M, A_k)$, where (P, C_s, M) are the OEM's decision variables and $k = 0,1,2$. In this way, the manufacturer's profits for each of the customer's possible actions are shown respectively in Eqs. (5), (6) and (7).

$$\Pi_{OEM}(P, C_s, M, A_0) = 0 \quad (5)$$

$$\Pi_{OEM}(P, C_s, M, A_1) = P - \sum_{j=1}^{M} C_r N(W_j) - \theta_1 T_{ov_1} - \theta_2 T_{ov_2} \quad (6)$$

$$\Pi_{OEM}(P, C_s, M, A_2) = N_{f2}(C_s - C_r) - \sum_{j=1}^{M} N(W_{1j}) C_r - \theta_1 T_{ov_1} \quad (7)$$

The manufacturer will choose the combination P, C_s, M that maximizes its expected profit, taking into account the customer's optimal strategy A^*.

3.5 Assumptions

In order to make the model manageable, we consider some assumptions:

I. The times between failures are random variables. The time to first failure follows a Weibull distribution, as seen in [16].
II. The times to repair are exponentially distributed with parameter μ;
III. The OEM has only one service channel, ie, they are capable of processing one unit at a time;
IV. The equipment's failures are critical. Moreover, the manufacturer carries out just corrective maintenance interventions;
V. The manufacturer and the customer have complete information about the model's parameters, which implies that the leader is aware about the customer's risk parameter and the hospital knows the equipment reliability;
VI. The equipment is a repairable system subject to imperfect repair. The probabilistic modelling used at the failure-repair process corresponds to the GRP;

It is important to note that since the repair capacity is fixed, greater values of M tend to increase waiting time for the equipment in queue, consequently resulting in greater chances of penalties for the OEM.

3.6 Hospital's Optimal Strategy

The customer's expected utility U is derived from two random variables (X_{ij}, Y_{ij}), the customer's decision A_k, and the pricing structure ($\mathrm{P}^{\max}$; C_s^{max}) imposed by the OEM. Given the assumptions of Section **Erro! Fonte de referência não encontrada.**, the expected utilities for each decision are given by eqs. (8), (9) and (10) obtained by using the eqs. (2), (3) and (4).

$$E[U(A_0, P, M, C_s)] = 0 \quad (8)$$

$$E[U(A_1, P, M, C_s)] = \frac{1}{\delta}\left\{1 - exp[\delta\ (P)]\ E\left[exp\left(-\delta\left(R\,T_{op} + \theta_1 T_{ov_1} + \theta_2 T_{ov_2}\right)\right)\right]\right\} \quad (9)$$

$$E[U(A_2, P, M, C_s)] = \frac{1}{\delta}\left\{1 - \exp[\delta]\ E\left[\exp\left(\delta\left(-R T_{op} - \theta_1 T_{ov_1} + N_{f2} C_s\right)\right)\right]\right\} \quad (10)$$

For a determined (P,C_s) proposed by the manufacturer, the customers analyze their three expected utilities and choose the option that will return the highest payoff.

Other point to emphasize corresponds to the reservation prices from the hospitals to the maintenance actions (P^{max}, C_s^{max}), i.e., the price at or below which consumer will demand one unit of the good. Those reservation prices will affect the decision A_k to be taken. As an example, at the case that the prices charged by the OEM becomes greater than the reservation prices of the healthcare institutions ($P > P^{max}$; $C_s > C_s^{max}$), then the equipment is not purchased (A_0). If the EW prices becomes superior than P^{max}, while C_s^{max} is not reached ($P > P^{max}$; $C_s \leq C_s^{max}$), the hospital chooses (A_2). Analogously, when $P \leq P^{max}$; $C_s > C_s^{max}$, the hospital chooses A_1.

Those reservation prices are directly linked to the expected number of failures of the equipment, and the number of units to be repaired. So they are fairly influenced by the repair structure. Considering imperfect repair, it is expected that similar devices will fail lass than in the context presented by [15], who considered minimal repairs. So, the changing of the failure structure may represent a change of results and consequently a change of strategies. The mathematical approach used to reproduce the imperfect repair is the GRP, a Monte Carlo based methodology that allows to model every type of repair, and bring more flexibility to this paper to represent better situations found in reality.

In addition, the formulation describes a queue with finite population M. The times to the first failure of each equipment follow a Weibull distribution, the rest of the failures are governed by a GRP. The service team can repair one unit at a time with constant average rate, they return the equipment to operational state increasing the reliability but not reverting as good as a new condition (imperfect repair). If there is more than one failed unit, a queue following a FCFS is generated.

Given the hospital's reservation prices and the pricing structure imposed by the OEM, the EW model can be seen on its extensive form, as a sequential game tree. **Erro! Fonte de referência não encontrada.** shows all possible decisions for the players. Therefore, determine the hospital's reservation prices is essential in finding the OEM's expected profit, once the pricing structure have extreme influence on the healthcare's institutions decision.

3.7 OEM's Optimal Strategy

Since the OEM is considered to be risk neutral, its optimal strategy corresponds to the pricing structure that maximizes their expected profit (P^{max}; C_s^{max}). Also, the OEM sets the optimal number of customers. To choose its optimal strategy, the OEM must compare the expected profit of each service modality, varying the number of customers M.

Considering assumption V in Section **Erro! Fonte de referência não encontrada.**, we conclude that the OEM has knowledge about the hospital's reservation prices. Then, the OEM builds a structure that captures all the consumer surplus, making the manufacturer behave as a monopolist. This scenario implies in the maximization of the producer surplus.

To find those reservation prices we must use eqs. (9) and (10). In order to determine P^{max} we must equalize eq. (9) to zero, and then isolate P. Thus, the reservation price of the EW can be given in (11). For calculating C_s^{max} we follow the same idea for P^{max}, now using eq. (10). However, since it's impossible to isolate C_s, we use a numerical method to find its expected value. Eq. (12) shows C_s^{max} equilibrium equation.

$$P^{max} = -\frac{1}{\delta} \ln E\left[\exp\left(-\delta\, R\, T_{op}\right)\, \exp\left(-\delta\, \theta_1\, T_{ov_1}\right)\, \exp\left(-\delta\, \theta_2\, T_{ov_2}\right)\right] \tag{11}$$

$$\delta\, + \ln E\left[\exp\left(-\delta\, R\, T_{op}\right)\, \exp\left(-\delta\, \theta_1\, T_{ov_1}\right)\, \exp\left(\delta\, N_{f_2}\, C_s^{max}\right)\right] = 0 \tag{12}$$

Therefore, the OEM will always set the price of its best option as the hospital's maximum willingness to pay, while the price will be superior than the hospital's maximum willingness to pay for the option that is not desired. By doing this, the hospital is induced to choose the option that maximizes the OEM's profit.

3.8 Queueing Simulation

As seen in section **Erro! Fonte de referência não encontrada.**, the presented formulation describes a queuing system with finite population (M), where arrivals (failures) and departures (completions of repairs) follow a Generalized Renewal Process due to the units being imperfectly repaired. Due to assumption II in section **Erro! Fonte de referência não encontrada.**, the times to repair are exponentially distributed. If there is more than one failed unit at a time, the queue follows a FCFS rule.

In order to find the optimal reservation prices P^{max} and C_S^{max}, it is necessary to simulate the alternating failure-repair process considering a G/M/1 queuing system. We used a DES approach due to the fact that simulation is able to realistically represent the behavior of a system of interest, allowing us to model and solve problems that would otherwise be considered intractable or too complex ([30]). The process described in this paper was simulated via a C++ program, thus allowing for estimation of failure and repair times, along with the estimation of other queue importance measures and model's metrics.

After the queue is simulated, customer's optimal strategy must be defined. To this end, we estimate the reservation prices for each option offered by the OEM, which are given by eqs. (11) and (12), also using the data from many replications of the simulation. Note that the amount that customers are willing to pay for each option depends on the magnitude of their risk aversion δ. The complete process used to find the optimal number of customers and their respective reservation prices is given in Figure 1, and is more detailed in **Erro! Fonte de referência não encontrada.**.

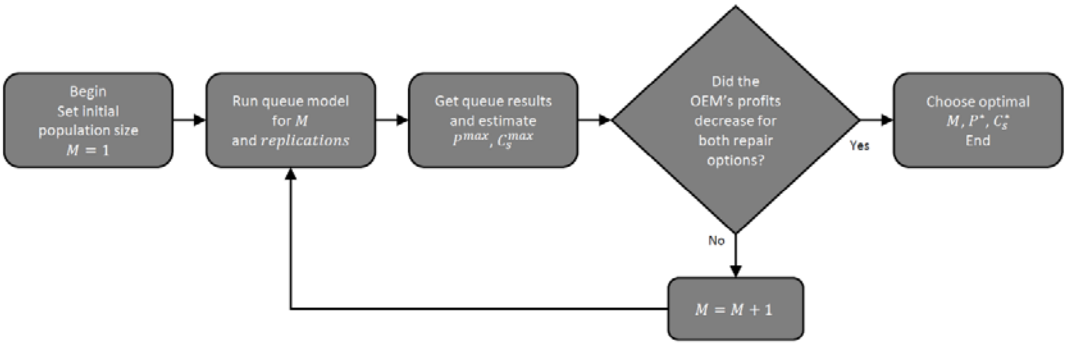

Figure 1 – Diagram of the optimization process presented in this paper

4. APPLICATION EXAMPLE

4.1 Model's Parameters

An application example was proposed by using a failure database of an Angiograph. This device is technology-intensive, and supports the treatment and diagnosis of cardiovascular diseases. Through the angiography, a radio-opaque fluid is injected into the blood vessels to the identification of its anatomy using radiation. We can define that the Angiograph is to the cardiovascular mapping as the ray-x are to mapping the osseous system.

The first step for our analysis was to take 38 critical failures records of the equipment and their respective times to repair from an available database. Considering that the equipment suffers imperfect repair and to reproduce the queuing system, then we need to use a simulation-based solution. To simulate the arrival times, following a GRP, we must at first obtain the MLEs with the procedure described by [16]. The optimal MLE's values considering the assumption of imperfect repair were for $\hat{\alpha}$, $\hat{\beta}$ and $\hat{q}$ respectively: 1351.83, 1.658 and 0.097.

In order to validate the estimators, a DES of times to failure was performed. The first 38 times to failure $X[j]$ were simulated using the MLEs found above, following the procedure described by [16], as mentioned in section **Erro! Fonte de referência não encontrada.**. Afterwards, such failure times were compared to the real failure data. Figure 2 shows the comparison between those two curves.

As expected, the predictions of the GRP model (considering imperfect repair assumption), show close agreement with the real collected data, which attest that the simulation model is suitable. For the application example, we will use the parameters shown in Table 1.

The inputs given above were used feed the stochastic process described in diagram presented in Figure 1 is executed, varying the number of M, P^{max} and C_s^{max} are estimated for each number of equipment (M), ultimately arriving at the scenario with the greatest payoff for the OEM.

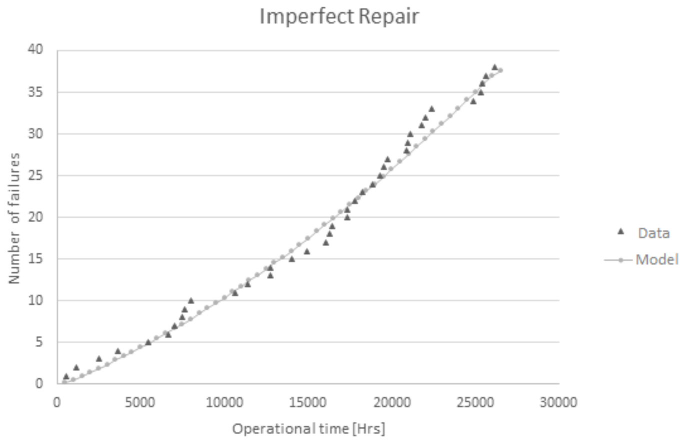

Figure 2 – Comparison between real and simulated times to failure (imperfect repair assumption)

Table 1 – Parameters for the application example

Angiograph sale price (C_b)	\$ 1,476.49 (10^3)
Revenue per time (R)	\$ 0.094 (10^3) h^{-1}
Cost of the repair (C_r)	\$ 2.5 (10^3)
Mean repair rate (μ)	0.2h^{-1}
Penalty per time (base warranty) (θ_1)	\$1 (10^3) h^{-1}
Penalty per time (under extended warranty) (θ_2)	\$3 (10^3) h^{-1}
Hospital's risk aversion (δ)	0.1
Period of the game (W).	2 years = 17520h
Maximal time to repair the equipment under EW (τ)	12h

By using those parameters and the GRP MLE's, the optimal number of equipment, and hospital's reservation prices found were respectively: $M^* = 47$, $\mathrm{P}^{\max}$ = **\$**183,991 and C_s^{max} = 8,550. As it may be seen, the price of hiring an EW is near the range between 5% and 12% of the purchase price, which is consistent to what happens in the clinical environment ([3]). The OEM's expected profit for the options (A_1, A_2) are respectively $E[\Pi_{OEM}(A_1)]$ = 2,979,171 and $E[\Pi_{OEM}(A_2)]$ = 2,155,601. The Nash Equilibrium occurs when the OEM sets P^* = $\mathrm{P}^{\max}$ and C_s^{max} > C_s^{max}, returning an expected profit of \$2,979,171. Finally, the optimal choice for the client is A_1 and the expected utility for the customer is zero, as the manufacturer is able to extract all the consumer's surplus ([13]).

4.2 Sensitivity Analysis

For better understanding about how sensitive the model is with regard to the parameters and repair assumptions, we present a sensitivity analysis.

First, the assumption of imperfect repair is relaxed and the perfect and minimal repairs are considered to identify output changes and possible alteration on the players' optimal strategies. Considering perfect repair, the simulated values for M^*, $\mathrm{P}^{\max}$ and C_s^{max} were 43; \$183,981; \$8,628 respectively. The manufacturer's optimal strategy is given by $P^* = \mathrm{P}^{\max}$ e $C_s > C_s^{max}$. To this configuration, the client chooses the option A_1 and the OEM's expected profit is \$2,377,056. By minimal repair assumption, the values for M^*, $\mathrm{P}^{\max}$ and C_s^{max} were 44; \$183,735 and \$7,735 respectively. The manufacturer's optimal strategy is given by $P^* = P^{max}$ e $C_s^* > C_s^{max}$. The customer's optimal strategy is given by A_1 and the manufacturer's expected profit is \$2,695,579. Table 2 summarizes all the results to each repair structure along the EW period (2 years).

Moreover, it is important to point out two other aspects: the variation of the hospital's maximum willingness to pay for single repair and the OEM's expected profit according to variations on the repair structure. By changing imperfect to perfect repair, C_S^{max} increases 0.9%. And the changing imperfect to minimal repair, C_S^{max} decreases 9,53%. That behavior shows that the client's maximum willingness to pay for on-call services decreases the more the equipment fails, therefore, he is more likely to get the EW. The OEM's payoff, in turn, by changing the assumption of imperfect to minimal repair, decreases 9,52%. A change between imperfect repair to perfect repair, represents a decline of 20.21%. It attests that, the assumption of the stochastic process is a fundamental decision about their payoff. A misguided assumption about the failure behavior of the equipment can represent a loss of $602,115 for the OEM.

Table 2 – Effect of repair hypothesis on optimal solution

Repair hypothesis	M^*	P^{max} $	C_S^{max} (10^3)$	A^*	E[π] $
Minimal Repair	44	183,735	7,735	A_1	2,695,579
Imperfect Repair	47	183,991	8,550	A_1	2,979,171
Perfect Repair	43	183,981	8,628	A_1	2,377,056

5. CONCLUDING REMARKS

In this paper, a decision model for an Extended Warranty involving a hospital and the OEM was proposed. For modelling this situation and determining the players' optimal strategies, a Stackelberg Game formulation was employed, with the OEM being the leader and the hospital being the follower. This situation is commonly found in the market of technology-intensive equipment, which is characterized by a greater bargaining power for the manufacturer since he is the only part capable of performing maintenance interventions adequately.

In order to approximate the problem to a more realistic context, we considered that the equipment is subject to imperfect repairs, and to model this issue, two approaches were joined, the GRP and queuing theory. Additionally, an application example was presented with real failure data of an angiograph to determine the optimal strategies for each player and demonstrate applicability of the model. Furthermore, we perform a series of sensitivity analyses by showing how model results and players' strategies behave under different repair types, and under variations on several of the model's parameters.

The present approach can be extended to cover the following situations:

- Consider asymmetry of information by employing a principal-agent formulation ([31], [32]).
- A dynamic Stackelberg Game with a greater time horizon to analyze the possibilities for renewal of the extended warranty, analyzing the behavior of the players during longer periods.
- Consider a heterogeneous market, with different profiles of customers, represented by different risk-aversion parameters.
- The number of service channels can be a decision variable to be optimized by the manufacturer;
- The model Kijima type II can be used to obtain GRP MLE's and simulate equipment failures. Those models (Kijima type I and II) can be compared to verify which is best given a dataset.
- By incorporating other approaches as seen in [25] to model GRP.
- A preventive maintenance policy can be incorporated to the model.

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