Analysis of Extended Warranties: an Approach Based on Generalized Renewal Process and Priority Queues

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ABSTRACT

In this paper, a model for maintenance service contracts is developed based on a Stackelberg game formulation, considering the interaction between manufacturer, which acts as service provider, and customers, who decide whether to buy a device and which kind of service to hire. Customers are divided into two distinct classes: class 1 is composed by large organizations, which prefer higher equipment availability over cost; class 2 is formed by small organizations, which prefer to pay lower prices for services, even if equipment availability is compromised. Equipment failure-repair behavior follows a generalized renewal process and, since there are multiple devices, each bought by a different customer, a queue system is formed. To the best of authors' knowledge, this is the first Stackelberg game application to join a generalized renewal process and a queueing system. A discrete event simulation approach is proposed for the solution of the model and an application example is presented.

Keywords: maintenance service contracts; extended warranty; generalized renewal process; priority queues; stackelberg game.

1 INTRODUCTION

Product warranties provide protection to both OEM and equipment buyer, making post-sale support an important aspect for product sale (Murthy & Djamaludin [1]). In addition to product's base warranty, extended warranties also play an important role in the context of maintenance services, since a significant number of customers tend to purchase extra protection against failures (Lutz & Padmanabhan [2]). Indeed, without warranty, customers are exposed to risks such as of excessively expensive repairs, critical equipment failures, poor repair quality, and others (Damnjanovic & Zhang [3]).

In this work, we approach the problem of maintenance service contracts (MSC) for technology-intensive equipment, considering a market with two customer classes. We adopt a similar customer class division as proposed by Moura et al. [4]. In fact, class 1 is composed by large organizations, with high potential for revenue generation, having the need for high equipment availability, even if at a higher cost, while class 2 is formed by smaller organizations, which have considerable market share, but not as high as class 1; these organizations have preference for less expensive maintenance services, even if it means that device availability is harmed.

In this context, the OEM offers priority and nonpriority types of services, so that the different customer classes can hire adequate maintenance services for their needs. OEM has limited capability for simultaneous repairs, thus, when waiting in queue for a repair, failed equipment with priority service starts being repaired before failed equipment with nonpriority service. Then, OEM defines adequate service prices for the different available service options, while customers analyze these prices and make their decision on buying (or not) a device, and which type of service to hire. Due to their characteristics, priority services are more expensive, but allow equipment to return faster to operational state, while nonpriority services result in lower equipment availability, but cost less.

An important aspect of the MSC context is the interaction among different agents. On one hand, service providers and customers have conflicting objectives, since customers want high equipment availability, with fast repairs when devices fail. On the other hand, service providers might want to serve many customers to increase their revenue, forming queues, and thus reducing equipment availability. These conflicting interactions can be modeled by adoption of a game theory approach, where each agent's objectives can be modeled, and an equilibrium can be reached (Greve [5]).

A Stackelberg game (SG) approach can be employed to model the relationship between customers and service provider. The original SG formulation (Gibbons [6]) considers the interaction between two agents, a leader and a follower, where the leader acts first, then the follower observes the leader's decision in order to make its own. In the context of MSC, OEM if thought of a leader, since it is the only service provider available to the market, while customers are the followers, observing the prices and service options offered by the OEM, then deciding which option to hire. This formulation of SG for MSC was used by Murthy & Yeung [7], Murthy & Asgharizadeh [8], [9], Asgharizadeh & Murthy [10], Moura et al. [4], among other authors.

As previously mentioned, the present model considers that OEM offers maintenance services to several customers, where these customers can be from two different classes. The existence of multiple customers can be modeled with queueing theory, considering the interaction service capability and demand (Gross et al. [11]). Since customers may belong to two different classes, each with different preferences, a priority queue formulation can be employed, where priority customers receive better service, accepting to pay more for this benefit. In the field of MSC, Moura et al. [4] used a two-class priority queue for modeling a similar situation, considering complex medical equipment.

In a realistic scenario, complex technology-intensive equipment may suffer degradation over time. A generalized renewal process (GRP) (Yañez et al. [12]) can be considered to model such behavior, while also allowing for generalization by covering a broad range of scenarios. In addition to modeling increasing, constant, and decreasing failure rates, GRP can also cover different levels of repair effectiveness, such as imperfect, perfect, and minimal repairs. Thus, this approach allows for a flexible and realistic model, which can be used in a wide variety of situations. Santana et al. [13] modeled the MSC problem for a set of equipment subject to a GRP, resulting in a queueing model.

In this paper, we present an approach to consider multiple customers in two priority classes, where each device follows a GRP. A MSC model for technology-intensive equipment is presented, considering the strategies of manufacturer (service provider) and customers (device buyers) of each class. We propose a simulation-based approach for achieving a solution, modeling equipment subject to a generalized renewal process (GRP) and in a two-class priority queue. This effectively extends the approaches presented by Moura et al. [4] and Santana et al. [13].

The remainder of this paper is briefly described as follows: section 2 gives a detailed description of the problem, explaining features of the proposed model, and introducing the agents' decision problems; section 3 develops the solution for the proposed model; section 4 presents an application example; Finally, section 5 concludes remarks.

2 MODEL DESCRIPTION

2.1 Game Description

The OEM sells a device and offers different types of maintenance services, chosen by buyers at the moment of purchase. Buyers, also called customers, are organizations that intend to generate revenue by using the device. Due to their different profiles, customers can be classified into two different classes:

- Class 1: Big organizations, with considerable market share that require high equipment availability, even if it costs more;
- Class 2: Small organizations, with not as dominant market share as big organizations; this class prefers less expensive maintenance service, even if equipment availability is reduced.

A base warranty for the device is provided by the OEM; depending on the hired maintenance service, this base warranty can be extended. Customers decide whether to buy a device and, if they buy it, which service type to hire. Service options offered by the OEM are divided into priority and nonpriority services. Thus, customers decide whether to hire priority service, and whether to extend equipment warranty. According to the preferences of each customer class, class 1 customers always opt for one of the priority services when they purchase a device, while class 2 customers opt for nonpriority services.

In total, there are five possible customer strategies, with one option not to buy a device, two options for hiring priority services, and two alternatives for hiring nonpriority services. These strategies are given in Table 1.

Table 1 – Description of the possible customer strategies

Strategy	Description
A_0	Not buying a device. Therefore, customers and OEM have no costs and receive no revenue.

Strategy	Description
A_1	Buying a device with priority service and extended warranty. Device has price C_b, and includes a base priority warranty with duration T_1; extended warranty costs $P_w^{(1)}$ and has duration T_2; $T = T_1 + T_2$ is the total coverage period. During base and extended warranty, every failure is repaired by the OEM without additional cost to the customer; as a priority service, devices covered by this plan begin repair before queued nonpriority devices, but follow FCFS order when in queue with other priority devices. The OEM must return the device to operational state (i.e., finish repairing the unit) within a time $\tau_i^{(1)}$, otherwise a penalty must be paid to the customer. Let y be the time between a failure and completion of its repair. The penalty is proportional to the overtime, $y - \tau_i^{(1)}$, i.e., the time spent after the limit $\tau_i^{(1)}$. During base warranty, penalty is equal to $\theta_1^{(1)}\left(y - \tau_1^{(1)}\right)$, and during extended warranty, penalty is equal to $\theta_2^{(1)}\left(y - \tau_2^{(1)}\right)$, where $\theta_i^{(1)}$ is a priority penalty rate for warranty period i, with $i = 1$ corresponding to the base warranty, and $i = 2$ being the extended warranty period. Note that only class 1 customers hire this type of service.
A_2	Buying a device with nonpriority service and extended warranty. Device has price C_b, and includes a base nonpriority warranty with duration T_1; extended warranty costs $P_w^{(2)}$ and has duration T_2. During base and extended warranty, every failure is repaired by the OEM without additional cost to the customer; as a nonpriority service, failed units must wait for queued priority equipment to be repaired before repair starts, following FCFS among nonpriority queued devices. If the OEM does not return the device to operational state within a time $\tau_i^{(2)}$, a penalty must be paid to the customer; during base warranty, penalty is equal to $\theta_1^{(2)}\left(y - \tau_1^{(2)}\right)$; and during extended warranty penalty is equal to $\theta_2^{(2)}\left(y - \tau_2^{(2)}\right)$. Note that only class 2 customers hire this type of service.
A_3	Buying a device with priority service, and priority on-call service after warranty expiration. Device has price C_b and includes a base priority warranty with duration T_1; after expiration of base warranty, the customer receives on-call service for a duration T_2, paying $C_s^{(1)}$ for each repair. $T = T_1 + T_2$ is the total coverage period. During base warranty, every failure is repaired by the OEM without additional cost to the customer; if the OEM does not return the device to operational state within a time $\tau_1^{(1)}$, a penalty must be paid to the customer, equal to $\theta_1^{(1)}\left(y - \tau_1^{(1)}\right)$. After base warranty expiration, device is covered by priority on-call service, where the customer must pay a fixed price $C_s^{(1)}$ for each repair; during this period (after expiration of base warranty), no penalty is incurred due to delays in repairs. Only class 1 customers hire this type of service.
A_4	Buying a device with nonpriority service, and nonpriority on-call service after warranty expiration. Device has price C_b and includes a base nonpriority warranty with duration T_1; after expiration of base warranty, the customer receives on-call service for a duration T_2, paying $C_s^{(2)}$ for each repair. During base warranty, every failure is repaired by the OEM without additional cost to the customer. During base warranty, the OEM must return the device to operational state within a time $\tau_1^{(2)}$, otherwise a penalty must be paid to the customer, equal to $\theta_1^{(2)}\left(y - \tau_1^{(2)}\right)$. After base warranty expiration, device is covered by nonpriority on-call service, where the customer must pay a fixed price $C_s^{(2)}$ for each repair; during this period, no penalty is incurred due to delays in repairs. Only class 2 customers hire this type of service

Each agent's decision problems are explained in more detail in subsections 2.2 and 2.3; a brief description is also provided as follows. OEM must decide how many customers to serve ($M^{(1)}$ and $M^{(2)}$), as well as each service price to be charged ($P_w^{(c)}$ and $C_s^{(c)}$, with $c \in \{1,2\}$). Customers of both classes, given the prices of each service, must decide whether to buy a device and which service type to hire. Each customer buys at most one device, and customers are considered homogeneous inside each class, so that all customers in each priority class choose the same strategy. Solution to this problem is reached by backward induction (Osborne & Rubinstein [14]), where the customers' decision problem is solved first, then the OEM's.

Notice that, due to their preferences, class 1 customers choose among strategies A_0, A_1 and A_3, while class 2 customers choose among A_0, A_2 and A_4. Device generates revenue when operational. Class 1 customers can generate revenue of $R^{(1)}$ per operational hour, while class 2 customers generate $R^{(2)}$ per operational hour. Notice that when failed, equipment stops generating revenue, and thus the efficiency of maintenance services is extremely important for OEM and customers. When equipment spends too much time in failed state, customers generate less revenue, and OEM must pay more penalties.

OEM is risk-neutral and seeks to maximize its expected profit. Customers are risk-averse, thus aim to maximize their expected utility. Customer's utility function for a wealth w is given by Eq. (1) (Varian [15]), where γ is the customer's risk aversion parameter. When γ is high, customers tend to avoid strategies with high uncertainty, seeking more predictable strategies to diminish eventual losses.

$$U(w) = \frac{1 - e^{-\gamma w}}{\gamma} \tag{1}$$

2.2 Customer's Decision Problem

As described in subsection 2.1, customers choose among the five possible decision alternatives, aiming to maximize their expected utility. In order to find the expected utility, we first obtain customer's wealth (return) for each alternative. For strategy A_0, customer's wealth is equal to zero, as in Eq. (2); for strategies A_1 and A_2, customer's wealth is given in Eqs. (3) and (4), respectively; and for strategies A_3 and A_4, Eqs. (5) and (6) respectively indicate customer's wealth. For customer j of class c: $T_j^{(c)}$ is the total operational time during coverage time T ($T = T_1 + T_2$); $O_{i,j}^{(c)}$ is the overtime during coverage period i; and $N_{i,j}^{(c)}$ is the number of failures during coverage period i.

$$w_{A_0} = 0 \tag{2}$$

$$w_{A_1} = R^{(1)}T_j^{(1)} + \theta_1^{(1)}O_{1,j}^{(1)} + \theta_2^{(1)}O_{2,j}^{(1)} - P_w^{(1)} - C_b \tag{3}$$

$$w_{A_2} = R^{(2)}T_j^{(2)} + \theta_1^{(2)}O_{1,j}^{(2)} + \theta_2^{(2)}O_{2,j}^{(2)} - P_w^{(2)} - C_b \tag{4}$$

$$w_{A_3} = R^{(1)}T_j^{(1)} + \theta_1^{(1)}O_{1,j}^{(1)} - N_{2,j}^{(1)}\, C_s^{(1)} - C_b \tag{5}$$

$$w_{A_4} = R^{(2)}T_j^{(2)} + \theta_1^{(2)}O_{1,j}^{(2)} - N_{2,j}^{(2)}\, C_s^{(2)} - C_b \tag{6}$$

By substituting Eqs. (2)-(6) into Eq. (1), it is possible to obtain the equations for customer's utility for each strategy alternative. The respective utilities for strategies A_0-A_4 are given in Eqs. (7)-(11).

$$U(w_{A_0}) = 0 \tag{7}$$

$$U(w_{A_1}) = \frac{1}{\gamma}\left\{1 - \exp\left[-\gamma\left(R^{(1)}\, T_j^{(1)} + \theta_1^{(1)}\, O_{1,j}^{(1)} + \theta_2^{(1)}\, O_{2,j}^{(1)} - P_w^{(1)} - C_b\right)\right]\right\} \tag{8}$$

$$U(w_{A_2}) = \frac{1}{\gamma}\left\{1 - \exp\left[-\gamma\left(R^{(2)}\, T_j^{(2)} + \theta_1^{(2)}\, O_{1,j}^{(2)} + \theta_2^{(2)}\, O_{2,j}^{(2)} - P_w^{(2)} - C_b\right)\right]\right\} \tag{9}$$

$$U(w_{A_3}) = \frac{1}{\gamma}\left\{1 - \exp\left[-\gamma\left(R^{(1)}\, T_j^{(1)} + \theta_1^{(1)}\, O_{1,j}^{(1)} - N_{2,j}^{(1)}\, C_s^{(1)} - C_b\right)\right]\right\} \tag{10}$$

$$U(w_{A_4}) = \frac{1}{\gamma}\left\{1 - \exp\left[-\gamma\left(R^{(2)}\, T_j^{(2)} + \theta_1^{(2)}\, O_{1,j}^{(2)} - N_{2,j}^{(2)}\, C_s^{(2)} - C_b\right)\right]\right\} \tag{11}$$

Eqs. (12)-(16) show customer's expected utilities for strategies A_0-A_4, respectively, which are obtained by evaluating the expected values for Eqs. (7)-(11) respectively.

$$E\left[U(w_{A_0})\right] = 0 \tag{12}$$

$$E\left[U(w_{A_1})\right] = \frac{1}{\gamma}\left\{1 - \exp\left[\gamma\left(P_w^{(1)} + C_b\right)\right] E\left[\exp\left[-\gamma\left(R^{(1)}\, T_j^{(1)} + \theta_1^{(1)}\, O_{1,j}^{(1)} + \theta_2^{(1)}\, O_{2,j}^{(1)}\right)\right]\right]\right\} \tag{13}$$

$$E\left[U(w_{A_2})\right] = \frac{1}{\gamma}\left\{1 - \exp\left[\gamma\left(P_w^{(2)} + C_b\right)\right] E\left[\exp\left[-\gamma\left(R^{(2)}\, T_j^{(2)} + \theta_1^{(2)}\, O_{1,j}^{(2)} + \theta_2^{(2)}\, O_{2,j}^{(2)}\right)\right]\right]\right\} \tag{14}$$

$$E\left[U(w_{A_3})\right] = \frac{1}{\gamma}\left\{1 - \exp\left[\gamma\, C_b\right] E\left[\exp\left[-\gamma\left(R^{(1)}\, T_j^{(1)} + \theta_1^{(1)}\, O_{1,j}^{(1)} - N_{2,j}^{(1)}\, C_s^{(1)}\right)\right]\right]\right\} \tag{15}$$

$$E\left[U(w_{A_4})\right] = \frac{1}{\gamma}\left\{1 - \exp\left[\gamma\, C_b\right] E\left[\exp\left[-\gamma\left(R^{(2)}\, T_j^{(2)} + \theta_1^{(2)}\, O_{1,j}^{(2)} - N_{2,j}^{(2)}\, C_s^{(2)}\right)\right]\right]\right\} \tag{16}$$

SG assumes complete and perfect information between the players; thus, after the OEM defines how many customers to serve ($M^{(c)}$) and service prices ($P_w^{(c)}$ and $C_s^{(c)}$), customers of both classes can estimate their expected utilities, since customers and OEM can estimate $T_j^{(c)}$, $O_{i,j}^{(c)}$ and $N_{i,j}^{(c)}$, which are the stochastic components of these equations. However, exact analytical estimation of these three metrics is intractable due to the model's characteristics, such as the assumption of a two-class priority GRP queue (Moura et al. [4]), and the assumption of risk-averse customers (Ashgarizadeh & Murthy [16]). Given the difficulties imposed by these considerations, the proposed model resorts to a discrete event simulation (DES) approach, as described in more detail in section 3. Using the DES approach, the values of $T_j^{(c)}$, $O_{i,j}^{(c)}$ and $N_{i,j}^{(c)}$ can be obtained, making possible to evaluate present model's equations.

2.3 Manufacturer's Decision Problem

The OEM seeks to maximize its expected profit, which depends on customers' decisions. For each customer decision alternative, OEM's profit can be found by using Eqs. (17)-(21), where C_r is the average cost spent by the OEM on execution of each repair. For each customer of class c who chooses extended warranty (strategy A_1 or A_2), the OEM receives $P_w^{(c)}$ for selling the extended warranty but must pay for all repairs during coverage period T, as well as for penalties during the base and extended warranty periods. When a customer of class c choses not to extend the warranty (strategy A_3 or A_4), OEM must pay for failures and penalties during base warranty but receives a payment of $C_s^{(c)}$ for every failure after base warranty expiration (although repair cost of C_r is still incurred to the OEM).

$$\pi_{A_0} = 0 \tag{17}$$

$$\pi_{A_1}\left(P_w^{(1)}, M^{(1)}\right) = M^{(1)} P_w^{(1)} - C_r \sum_{j=1}^{M^{(1)}} N_j^{(1)} - \theta_1^{(1)} \sum_{j=1}^{M^{(1)}} O_{1,j}^{(1)} - \theta_2^{(1)} \sum_{j=1}^{M^{(1)}} O_{2,j}^{(1)} \tag{18}$$

$$\pi_{A_2}\left(P_w^{(2)}, M^{(2)}\right) = M^{(2)} P_w^{(2)} - C_r \sum_{j=1}^{M^{(2)}} N_j^{(2)} - \theta_1^{(2)} \sum_{j=1}^{M^{(2)}} O_{1,j}^{(2)} - \theta_2^{(2)} \sum_{j=1}^{M^{(2)}} O_{2,j}^{(2)} \tag{19}$$

$$\pi_{A_3}\left(C_s^{(1)}, M^{(1)}\right) = \left(C_s^{(1)} - C_r\right) \sum_{j=1}^{M^{(1)}} N_{2,j}^{(1)} - C_r \sum_{j=1}^{M^{(1)}} N_{1,j}^{(1)} - \theta_1^{(1)} \sum_{j=1}^{M^{(1)}} O_{1,j}^{(1)} \tag{20}$$

$$\pi_{A_4}\left(C_s^{(2)}, M^{(2)}\right) = \left(C_s^{(2)} - C_r\right) \sum_{j=1}^{M^{(2)}} N_{2,j}^{(2)} - C_r \sum_{j=1}^{M^{(2)}} N_{1,j}^{(2)} - \theta_1^{(2)} \sum_{j=1}^{M^{(2)}} O_{1,j}^{(2)} \tag{21}$$

However, since customers decide based on service prices defined by the OEM ($P_w^{(c)}$ and $C_s^{(c)}$), as well as the number of customers served ($M^{(c)}$), the OEM can influence customer's decision. Therefore, OEM's total profit π is the sum of the profits π_{A_k} which result from each customer class. Given the values of $P_w^{(c)}$, $C_s^{(c)}$ and $M^{(c)}$ for each class c, the OEM estimates its expected profit for each possible customer strategy by evaluating the expected values of Eqs. (17)-(21). Thus, in order to maximize its profit, the OEM needs to choose adequate values for $P_w^{(c)}$, $C_s^{(c)}$ and $M^{(c)}$, using backward induction. This process is described in section 3. Also, for the OEM's decision problem to be solved, the values of $T_j^{(c)}$, $O_{i,j}^{(c)}$ and $N_{i,j}^{(c)}$ are also needed. As already mentioned, these are obtained through a DES approach, described as well in section 3.

2.4 Failure-Repair Behavior

A simplified representation of an example system following this game description is given in Figure 1. The first timeline, indicated by a star on its left, denotes a priority device, while the three remaining timelines correspond to three nonpriority devices; in this example, a single repair crew is available, i.e., a single device can be repaired at a time. Equipment that fails while a device is being repaired must wait for completion of the current repair. When a repair is finished, the next device in queue has its repair started; a nonpriority device can only start being repaired when there is no failed priority device in queue.

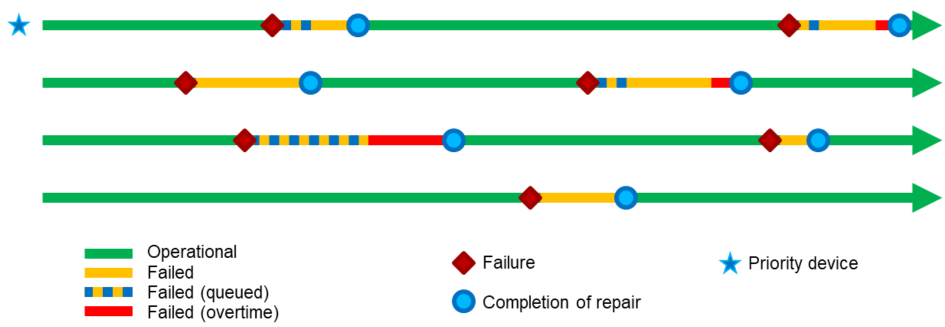

Figure 1 – Simplified representation of the two-class priority GRP queue model

Each device, in respect to its failure behavior, follows a Kijima Type I Generalized Renewal Process (GRP) (Moura et al. [17]). Since several devices may be sold, bought by customers of both classes, equipment under repair service coverage forms a two-class priority GRP queue, a GRP/Markovian/$m/\infty/M^{(1)}+M^{(2)}$/2-class-FCFS. Thus, failure times for each device follows a GRP-Weibull distribution with scale parameter α, shape parameter β, and rejuvenation parameter q. Repair times follow an exponential distribution with rate μ, and may be imperfect, perfect or minimal, depending on the value of q.

3 MODEL SOLUTION

3.1 Estimation of Service Prices

Since the model here developed considers complete and perfect information, the OEM can predict the decision of customers, which, in turn, decide based on the observed service prices and total number of customers chosen by OEM. Thus, the OEM acts first, having to choose values for $P_w^{(c)}$, $C_s^{(c)}$ and $M^{(c)}$ to maximize its own expected profit $E[\pi]$. In order to do this, the OEM analyzes customers' decision, by applying the process of backward induction.

Given a number of customers $M^{(c)}$, the OEM needs to find $P_w^{(c)*}$ and $C_s^{(c)*}$, which are the respective values of $P_w^{(c)}$ and $C_s^{(c)}$ that make customers of class c indifferent to all strategies, i.e., by making $P_w^{(c)} = P_w^{(c)*}$ and $C_s^{(c)} = C_s^{(c)*}$, customers would expect the same value of utility for every strategy. Since the expected utility of strategy A_0 is zero, this means that when $P_w^{(c)} = P_w^{(c)*}$ and $C_s^{(c)} = C_s^{(c)*}$, customers' expected utilities are all equal to zero, so that Eqs. (22) and (23) are true. Note that $P_w^{(c)*}$ and $C_s^{(c)*}$ are the respective maximum prices a customer of class c is willing to pay for extended warranty and on-call repairs.

$$E\left[U\left(w_{A_0}\right)\right] = E\left[U\left(w_{A_1}\right)\right] = E\left[U\left(w_{A_3}\right)\right] = 0 \tag{22}$$

$$E\left[U\left(w_{A_0}\right)\right] = E\left[U\left(w_{A_2}\right)\right] = E\left[U\left(w_{A_4}\right)\right] = 0 \tag{23}$$

In order to obtain $P_w^{(c)*}$ and $C_s^{(c)*}$, we first equate expected utilities among each customer class to zero, that is, equate each of Eqs. (12)-(16) to zero. This allows us for obtaining relations for estimating $P_w^{(c)*}$ and $C_s^{(c)*}$. $P_w^{(c)*}$ is found by equating Eqs. (13) or (14) to zero, and then solving to $P_w^{(c)}$, which results in Eq. (24). Analogously, Eqs. (15) and (16) are equated to zero for finding $C_s^{(c)*}$. Even though a closed form equation cannot be found, we obtain Eq. (25), which yields $C_s^{(c)*}$ when solved under numerical methods. Also, notice that once more the stochastic values of $T_j^{(c)}$, $O_{i,j}^{(c)}$ and $N_{i,j}^{(c)}$ become necessary for evaluation of equations.

$$P_w^{(c)*} = -C_b - \frac{1}{\gamma} \ln E\left[\exp\left[-\gamma\left(R^{(c)}\, T_j^{(c)} + \theta_1^{(c)}\, O_{1,j}^{(c)} + \theta_2^{(c)}\, O_{2,j}^{(c)}\right)\right]\right] \tag{24}$$

$$\gamma\, C_b + \ln E\left[\exp\left[-\gamma\left(R^{(c)}\, T_j^{(c)} + \theta_1^{(c)}\, O_{1,j}^{(c)} - N_{2,j}^{(c)}\, C_s^{(c)*}\right)\right]\right] = 0 \tag{25}$$

3.2 Optimal Solution

Note that, given the number of customers in each class ($M^{(1)}$ and $M^{(2)}$), the OEM finds each customer's maximum willingness to pay for extended warranty and on-call repairs, as shown above. Thus, so that OEM's profit is maximized, it is possible to substitute the values of $P_w^{(c)*}$ and $C_s^{(c)*}$ into the equations for π_{A_k}, resulting in the profit each possible customer decision alternative would yield. Then, the OEM compares these profits, finding which set of customer decisions (combination of strategies of class 1 and 2) is the best. Then, it is possible to define $P_w^{(c)}$ and $C_s^{(c)}$, inducing customers to choose the best strategies for the OEM. This is done as listed below, also given in summarized form in Table 2.

- $P_w^{(c)} > P_w^{(c)*}$ and $C_s^{(c)} > C_s^{(c)*}$: customers of class c choose strategy A_0, deciding not to buy a device, since both prices charged are higher than their maximum willingness to pay;
- $P_w^{(c)} = P_w^{(c)*}$ and $C_s^{(c)} > C_s^{(c)*}$: customers of class c choose strategy A_1 (when $c = 1$) or A_2 (when $c = 2$), extending their warranty, since the price for on-call repairs are higher than what they accept to pay;

- $P_w^{(c)} > P_w^{(c)*}$ and $C_s^{(c)} = C_s^{(c)*}$: customers of class c choose strategy A_3 (when $c = 1$) or A_4 (when $c = 2$), not extending their warranty, since the price for extending the warranty is higher than their maximum willingness.

Table 2 – Customer strategy for possible price combinations

Prices	$P_w^{(c)} > P_w^{(c)*}$	$P_w^{(c)} = P_w^{(c)*}$
$C_s^{(c)} > C_s^{(c)*}$	A_0	A_1 or A_2
$C_s^{(c)} = C_s^{(c)*}$	A_3 or A_4	A_1/A_2 or A_3/A_4

The number of customers to be served in each class is also defined by the OEM for maximization of its profit. The procedure shown above for definition of service prices assumes that these numbers have already been defined. For the optimal number of customers in each class, the OEM evaluates their expected profit for every possible combination of numbers of customers in the two classes. Finally, the combination of $M^{(1)}$ and $M^{(2)}$ that results in the greatest OEM's profit is chosen.

3.3 Simulation Approach

Discrete event simulation (DES) is the process of replicating stochastic processes by employing modeling with variables and discrete events (Ross [18]). DES makes possible the replication of real complex processes, allowing for better understanding, easier and more robust decision making. When dealing with models, where analytical solution is not possible or is too complex, DES becomes especially useful (Zio [19]). In our case, DES is used for finding the values of the stochastic variables $T_j^{(c)}$, $O_{i,j}^{(c)}$ and $N_{i,j}^{(c)}$, needed for obtaining a model solution.

3.3.1 Priority GRP Queue Simulation Algorithm

As previously stated, obtainment of values for $T_j^{(c)}$, $O_{i,j}^{(c)}$ and $N_{i,j}^{(c)}$, which result as metrics of the GRP/Markovian/$m/\infty/M^{(1)} + M^{(2)}$/2-class-FCFS queue, requires the application of simulation methods, due to analytical limitations in face of the model's complexity. Thus, a DES algorithm for simulating such model was developed, and is presented in Figure 2. A description of the algorithm is given in the following paragraphs.

Initially, in step 1 of the algorithm, simulation time is set to $t = 0$, and, the time of next completion of repair is initially set to $t_d = \infty$ so that the first event is never a repair. First, failure times for every device are generated using the GRP-Weibull distribution; t_{a_1} and t_{a_2} are respectively set to the earliest times of failures among devices of each class. Then, t_{a_1}, t_{a_2} and t_d are compared, so that the next event can be identified. When $t_{a_c} < t_d$, the next event is a failure of a device of class c; if $t_d < t_{a_1}$ and $t_d < t_{a_2}$, then the next event is a completion of repair. When an event occurs, simulation time is updated to the event time, and necessary actions are executed, updating variables and saving results whenever necessary.

When a failure occurs, step 2.1.1 in the algorithm, the number of failed devices is incremented; the respective device's number of failures for the current coverage period is also incremented; failure time is saved, so that downtime and overtime can later be calculated. If there is a free server at failure time, then the device is immediately repaired; thus, a repair time is generated, allowing for the calculation of repair completion time t_d. The difference between repair completion time and failure time is used for calculating downtime and overtime, which are then added to the respective device's variables for these values. Finally, a new failure time for class c is set, which is the value of the earliest failure time among the remaining operational equipment in that class; if all equipment of class c is in failed state, then $t_{a_c} = \infty$.

If a repair is completed, step 2.1.2 in the algorithm, the number of failed devices is decremented. A failure time for the device just repaired is generated and compared to the earliest failure time among the remaining operational equipment of class c; if the generated time is earlier than the others, t_{a_c} is set to that time. If there is any failed priority device in queue, the next priority device in queue begins its repair; if there are only nonpriority devices in queue, the next nonpriority device in queue begins its repair; if there are no failed equipment in queue, then t_d is set to infinity. If a device from the queue is chosen to begin its repair, its repair time is generated, t_d is calculated, and its downtime and overtime registers are updated, similarly as in the previous paragraph.

Inputs: α, β, q, μ, m, $\tau_1^{(1)}$, $\tau_2^{(1)}$, $\tau_1^{(2)}$, $\tau_2^{(2)}$, T_1, T_2, $M^{(1)}$, $M^{(2)}$

1. **Initialization**
   - 1.1. Generate first failure time for each device and set t_{a_1} and t_{a_2} equal to the earliest failure time of each customer class
   - 1.2. For convenience, set $t_d = \infty$
   - 1.3. Set initial time: $t = 0$
2. **Simulation**
   - 2.1. Is the next event a failure ($t_{a_c} \leq t_d$) or a completion of repair ($t_d < t_{a_c}$)?
     - 2.1.1. Failure of device j of class c (only if $t_{a_c} < T$)
       - Update current time $t = t_{a_c}$
       - Store failure information
         - Increment number of failures for device j for current warranty period i ($N_{i,j}^{(c)}$)
         - Increment number of failed devices
         - Store failure time
       - If there is at least one free server: begin repair immediately
         - Generate repair duration r and set $y = r$
         - $t_d = t + y$
         - Increase downtime by y for device j for current warranty period i ($D_{i,j}^{(c)}$)
         - Store repair duration and departure time
         - If $y > \tau$, increase overtime by $y - \tau_i^{(c)}$ for device j in current warranty period i ($O_{i,j}^{(c)}$)
       - Set t_{a_c} to the earliest failure time among the remaining operational equipment in class c
     - 2.1.2. Completion of repair on device j in class c
       - Update current time $t = t_d$
       - Store repair information
         - Decrement number of failed devices
         - Generate a failure time t'_{a_c} for this device
         - If $t'_{a_c} < t_{a_c}$, then set $t_{a_c} = t'_{a_c}$
       - Are there any remaining failed devices?
         - Yes
           - Let u be the time spent in queue
           - Generate repair duration r and set $y = u + r$
           - $t_d = t + y$
           - Increase downtime by y for device j for current warranty period i ($D_{i,j}^{(c)}$)
           - Store repair duration and departure time
           - If $y > \tau$, increase overtime by $y - \tau_i^{(c)}$ for device j in current warranty period i ($O_{i,j}^{(c)}$)
         - No
           - Set $t_d = \infty$
   - 2.2. Is $t_{a_c} > T$ and the system empty?
     - 2.2.1. Yes: go to step 3.
     - 2.2.2. No: repeat step 2.
3. **Output generation:** for each device, during each warranty period, return the following measures:
   - 3.1. Number of failures ($N_{1,j}^{(c)}$, $N_{2,j}^{(c)}$)
   - 3.2. Downtime ($D_{1,j}^{(c)}$, $D_{2,j}^{(c)}$)
   - 3.3. Overtime ($O_{1,j}^{(c)}$, $O_{2,j}^{(c)}$)

Figure 2 – Priority GRP Queue DES algorithm

When $t_{a_c} > T$, no more failures will be simulated, since the next failure would occur after the coverage period. However, remaining failed devices are repaired, until the queue is empty. When the condition that $t_{a_c} > T$ and the queue is empty is met, simulation can be stopped. The outputs of the simulation are, for each customer in each class, the number of failures during each coverage period ($N_{i,j}^{(c)}$), downtime for each coverage period ($D_{i,j}^{(c)}$), and overtime for each coverage period ($O_{i,j}^{(c)}$). Notice that, operational time can be calculated from downtime by doing $T_{i,j}^{(c)} = T_i - D_{i,j}^{(c)}$ or $T_j^{(c)} = T - D_{1,j}^{(c)} - D_{2,j}^{(c)}$.

3.3.2 Optimization Algorithm

Figure 3 contains the algorithm used for solving the optimization problem presented in this work.

Inputs: α, β, q, μ, m, $\theta_1^{(1)}$, $\theta_2^{(1)}$, $\theta_1^{(2)}$, $\theta_2^{(2)}$, γ, $\tau_1^{(1)}$, $\tau_2^{(1)}$, $\tau_1^{(2)}$, $\tau_2^{(2)}$, $R^{(1)}$, $R^{(2)}$, C_b, C_r, T_1, T_2, $M^{(1)}$, $M^{(2)}$, n_{rep}

1. **Initialization**:
   1.1. Set initial population size ($z = 1$) and initial class combination ($M^{(1)} = z$, $M^{(2)} = 0$)
2. **Model execution**
   2.1. Run priority GRP queue simulation for the current population $M = M^{(1)} + M^{(2)}$ for the required number of replications n_{rep}
   2.2. Get queue measures from simulation
      2.2.1. Estimate $C_s^{(c)*}$ and $P_w^{(c)*}$
      2.2.2. Choose best strategy for each class
      2.2.3. Store OEM's profit and strategies
   2.3. Have all combinations of $M^{(1)}$ and $M^{(2)}$ been simulated?
      2.3.1. No:
         2.3.1.1. $M^{(1)} = M^{(1)} - 1$, $M^{(2)} = M^{(2)} + 1$
         2.3.1.2. Repeat step 2
      2.3.2. Yes:
         - Did OEM's profits decrease for both repair service options?
           - No
             - Increment population size $z = z + 1$
             - $M^{(1)} = z$, $M^{(2)} = 0$
             - Repeat step 2
           - Yes: go to step 3
3. **Results definition**
   3.1. Choose population size and strategies which resulted in highest OEM profit
   3.2. Define $C_s^{(c)}$ and $P_w^{(c)}$ accordingly

Figure 3 – Model optimization algorithm

At first, population size is set to $z = 1$, and customer class size is then defined as $M^{(1)} = z$ and $M^{(2)} = 0$. Then, step 2.1 is executed, which is actually the DES algorithm proposed previously in Figure 2; simulation is run for n_{rep} Monte Carlo replications. Simulation results are obtained, and the best strategies and OEM's profit are saved. Next, it is checked whether all possible combinations of $M^{(1)}$ and $M^{(2)}$ with $M^{(1)} + M^{(2)} = z$ are simulated. If not, $M^{(1)}$ is decremented, $M^{(2)}$ is incremented, and step 2 is repeated. After all combinations are simulated, the population size z itself is incremented, and step 2 is repeated. Finally, after all possible population sizes and combinations were tested, it is possible to define the optimal solution, which is the number of customers and strategies that result in the greatest OEM's profit. Based on these results, $P_w^{(c)}$ and $C_s^{(c)}$ can be defined.

4 APPLICATION EXAMPLE

4.1 Example Description

For demonstrating the present methodology, an application example is given in this section. Actual failure data from a technology-intensive medical device, fitting the general characteristics covered by the methodology, are here presented. The data are from an angiography device, used for imaging examinations with help of a contrast agent, allowing doctors to visualize blood vessels and blood flow (Dyro [20]; Suri & Laxminarayan [21]). Failures might cause diagnostic errors, imprecise readings and even prevent it from functioning at all. In this context, the customers are hospitals that intend to buy an angiography device. Class 1 customers are big hospitals, with a higher number of patients, while class 2 customers are smaller hospitals and clinics.

In order to simulate equipment behavior, we first estimate GRP parameters α, β and q by employing the maximum likelihood estimators (MLEs)-based method proposed by Yañez et al. [12]. Estimated values were $\hat{\alpha} = 1{,}351.83$ h, $\hat{\beta} = 1.658$, and $\hat{q} = 0.097$. These parameters were used as inputs for the application example. The GRP MLEs, as well as the remaining model's parameters used in the application example, are shown in Table 3.

Then, the methodology described throughout this text was applied, following the steps in Figure 3. Figure 4 shows the expected number of failure over time, using simulated data by considering $\hat{\alpha}$, $\hat{\beta}$ and $\hat{q}$, demonstrating close agreement with real data. The models' results are shown in the following subsection.

Table 3 – Parameters used for the application example

Description	Value	Description	Value
GRP-Weibull scale parameter (α)	1,351.83 h	Extended warranty duration (T_2)	8,760 h (1 year)
GRP-Weibull shape parameter (β)	1.658	Base penalty rate for class 1 ($\theta_1^{(1)}$)	\$ 1 (10^3) / h
Repair parameter q	0.097	Extended penalty rate for class 1 ($\theta_2^{(1)}$)	\$ 3 (10^3) / h
Repair rate (μ)	0.1 h^{-1}	Base penalty rate for class 2 ($\theta_1^{(2)}$)	\$ 0.5 (10^3) / h
Customer risk-aversion (γ)	0.1	Extended penalty rate for class 2 ($\theta_2^{(2)}$)	\$ 2 (10^3) / h
Equipment price (C_b)	\$ 1,476.49 (10^3)	Unpenalized time for class 1 ($\tau_1^{(1)} = \tau_2^{(1)}$)	24 h
Class 1 operational revenue ($R^{(1)}$)	\$ 0.1 (10^3) / h	Unpenalized time for class 2 ($\tau_1^{(2)} = \tau_2^{(2)}$)	36 h
Class 2 operational revenue ($R^{(2)}$)	\$ 0.096 (10^3) / h	OEM cost per repair (C_r)	\$ 2.5 (10^3)
Base warranty duration (T_1)	8,760 h (1 year)	Number of replications (n_{rep})	1,000,000

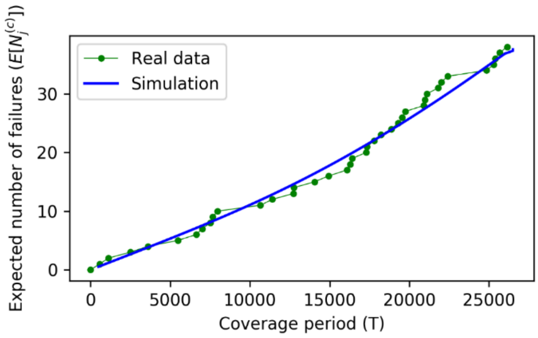

Figure 4 – Simulated expected number of failures over time, along with real failure data

4.2 Results

Model's main results for the application example are given in Table 4. On the optimal solution, the OEM sells devices to $M^{(1)} = 3$ class 1 customers and $M^{(2)} = 26$ class 2 customers, with a total of $M = 29$. Both class 1 and class 2 customers choose the extended warranty; thus, class 1 customers choose strategy A_1, while class 2 customers choose strategy A_2. Note that priority services have considerably higher prices, since priority customers suffer less downtime, and receive more compensation in the form of penalties when there is overtime. OEM's resulting expected profit is $E[\pi] = \$\ 2{,}584{,}038$.

Table 4 – Main results for the application example

Metric	Value
Number of priority customers ($M^{(1)}$)	3
Number of nonpriority customers ($M^{(2)}$)	26
Price of priority extended warranty ($P_w^{(1)*}$)	\$ 280,301
Price of nonpriority extended warranty ($P_w^{(2)*}$)	\$ 12,121
Price of priority on-call repair ($C_s^{(1)*}$)	\$ 192,846
Price of nonpriority on-call repair ($C_s^{(2)*}$)	\$ 8,833
Class 1 customers' strategy ($A^{(1)}$)	A_1
Class 2 customers' strategy ($A^{(2)}$)	A_2
OEM's expected profit ($E[\pi]$)	\$ 2,584,038

It is also interesting to analyze some queue indicators and performance measures; these are given in Table 5. Notice that priority devices spend 12.75% less time in failed state than nonpriority ones. Also, expected overtime for priority customers is higher than for nonpriority customers. This occurs because the unpenalized time $\tau^{(c)}$ is much stricter for priority customers, therefore, even though they receive better service, the time limit for the OEM to finish repairing priority equipment is much shorter, resulting in higher likelihood of overtime occurrence.

Table 5 – Main metrics and performance measures

Metric	Value
Total expected number of failures for a priority device ($E\left[N_j^{(1)}\right]$)	21.22
Total expected number of failures for a nonpriority device ($E\left[N_j^{(2)}\right]$)	21.21
Total expected downtime for a priority device ($E\left[D_j^{(1)}\right]$)	290.76 h
Total expected downtime for a nonpriority device ($E\left[D_j^{(2)}\right]$)	333.25 h
Total expected overtime for a priority device ($E\left[O_j^{(1)}\right]$)	43.79 h
Total expected overtime for a nonpriority device ($E\left[O_j^{(2)}\right]$)	35.05 h

It is possible to notice that, although priority service is considerably more expensive, it offers a better situation for class 1 customers, since downtime is lower than for nonpriority customers, and yet compensation with penalties is higher. This is also a reflection of the characteristics of each customer class, since class 1 customers want high equipment availability, even if it costs higher, while class 2 customers prefer to pay lower prices at the cost of worse equipment availability. Also, class 1 customers can generate more hourly revenue with their devices, which increases their willingness to pay for priority services.

5 CONCLUDING REMARKS

This paper presented an approach for modeling MSC of technology-intensive equipment, considering a market with customers in two different classes, one formed by big organizations that require high equipment availability, and the other with smaller organizations that prefer lower service prices. The OEM was considered risk-neutral and intended to maximize its expected profit, while customers were considered risk-averse and aimed to maximize their expected utilities.

Game theory was employed for modeling the interaction among service provider (OEM) and customers. Equipment followed a failure-repair behavior governed by a GRP, with virtual age conditioned Weibull distributed times until failures, and imperfect repairs; however, due to the existence of multiple devices, a priority queue system was formed. This required the usage of a discrete event simulation approach for finding the model's metrics and solutions.

Following a Stackelberg game formulation, perfect and complete information was assumed, so that OEM and customers have complete and perfect knowledge about each other's behavior and decisions, as well as perfectly knowing equipment reliability behavior. For instance, OEM could predict customers' decisions, using this information to maximize its own profit by backward induction. All customers of a given class were homogeneous, meaning that they make the same decision, whereas customers of different classes may decide differently.

An application example was presented, using real failure data from a technology-intensive medical equipment, along with some comments about model behavior. Other possible application scenarios include (but are not limited to) wind turbines, mining trucks (Dyamasius et al. [22]), and industrial robotic tools.

Although presented model overcame limitations of some existing methods, there is still room for improvement, which is concern of our ongoing research. Some possibilities for future modifications or extensions are: (i) Inclusion of preventive maintenance actions (Moura et al. [17]); (ii) Consideration of information asymmetry (Jin et al. [23]), so that OEM and customers cannot precisely predict each other's decisions and equipment behavior; (iii) Incorporation of two-dimensional warranty, considering effects such as time and usage into equipment reliability (Samatli-Paç & Taner [24]); and (iv) Allowing for renewal of warranty and/or extension of warranty after the moment of purchase.

ACKNOWLEDGMENTS

The authors would like to thank the National Council for Scientific and Technological Development – CNPq and Fundação de Amparo à Ciência e Tecnologia do Estado de Pernambuco – FACEPE for the financial support through research grants. This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

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