**Design of Maintenance Service Contracts via Principal-Agent Models**

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**ABSTRACT**

At the acquisition ofcomplex equipment such as electronic systems and medical devices, the buyeris usually not able to perform maintenance in-house. In these cases, maintenance actions are outsourced through a maintenance service contract (MSC) to external agents. In this paper, the owner of a complex equipment, called “principal”, intends to outsource maintenance services to a specialized service provider (agent). The equipment is subjected to imperfect repairs, and its times to failure are modeled by a Weibull-Generalized Renewal Process (GRP), a flexible statistic distribution. Several maintenance agents in the market are capable to perform imperfect repairs, however, these agents ate heterogeneous regarding their repair rates () which are distributed by a cumulative distribution function , on a bounded interval. Thequickness of the execution of the maintenance task is private information of the agent, which characterizes a scenario of asymmetric informationThis paper intends to maximize agent’s payoff by designing a contract under information asymmetry that transfer to the agent the best incentives (pricing and penalties) in order to stimulate agent’s proper self-selection. Due to the fact of not being possible to obtain analytical values of model’s random variables, a Discrete Event Simulation (DES) algorithm is implemented to simulate agent’s queuing system.Additionally, the continuous distribution will be approximated through DES by the use of random number generator functions.A Particle Swarm Optimization (PSO) method is employed to search for optimal solutions for the constrained problem. To demonstrate the applicability of the model, an application example is presented.

1. **INTRODUCTION**

The acquisition of assets for productive processes are managerial decisions that extend the time of purchasing, due to the impact on revenues generated by those devices and expenses such as operations and maintenance (O&M) costs. At the time of purchasing an equipment, a base warranty is usually offered by Original Equipment Manufacturer (OEM). This base warranty is tied to sales process to ensure manufacturer’s confidence in their product and, from customer’s perspective, it is an assurance that the product will perform satisfactorily over the warranty period [1].

For complex equipment such as electronic systems, medical devices (e.g., scanners and magnetic resonance imaging systems), wind turbines and photo-voltaic cells, customers are usually not able to perform maintenance in-house. In these cases, maintenance actions are carried out by an external party through a maintenance service contract (MSC) to, which can be the OEM or an outsourced agent [2].

Other benefits of outsourcing maintenance rather than performing it in-house were pointed out by [3]as i) access to high level experts and technology; ii) better service due to expertise of the service provider; iii) predictability of maintenance costs, since fixed price contracts prevent extremely high costs; iv) less capital investment for the customer; and v) managers can focus on core activities of the company. In this context, reliable systems are required to guarantee system safety and operation, avoiding unwanted costs such as machinery changes and unneeded maintenance. So, the modeling of repairable systems, through simulation and mathematical models, is important to analyze and predict the system behavior and to ensure reliability and maintenance efficiency

The problem of maintenance outsourcing between two firms can be modeled using game theory framework, in which one customer (the principal) owns a complex device, and delegates to the OEM or a third independent agent to take maintenance actions over a time length . The principal is not able to observe the action adopted by the agent, but s/he observes the corresponding result. Then, the principal plans to design a contract, with the best incentives to induce the agent to take the best actions from the principal’s viewpoint. We refer to this kind of problem as a principal-agent (PA) problem [4], where the unobserved agent’s action characterizes a situation of information asymmetry.

The exact cost of a task, the precise technology needed, the time of execution of this task or how good is the matching between the agent's intrinsic ability and technology are some examples of pieces of information which may remain private knowledgeof the agent. At this situation we say that exist asymmetric information or hidden knowledge [5]. A context of information asymmetry can me modeled as a principal-agent problem, in which one party, called agent, is contracted to take the best action from the other party (called principal) viewpoint. The interests between agent and principal are not aligned as the latter has less of information than the former.

At the best of authors knowledge, [6] and [7] were the first authors to use Stackelberg game (SG) to model MSC. They considered a perfect information game, in which all equipment units were identical regarding their reliability and the system, after a corrective maintenance (CM), returned to an as good as new condition. As a result, they found the optimal pricing structure, number of devices to perform maintenance and the number of service channels from agent’s perspective. Based on those papers, [8] designed a warranty contract applied into healthcare sector, in which the hospitals could belong to two priority classes. To belong to the class with a non-preemptive priority, hospitals would have to pay more. In order to determine the optimal pricing of the contract, pricing of on-call maintenance and number of hospitals to serve, a discrete event simulation (DES) algorithm was developed to represent a two-class Markovian queue. [9], despite modeling a queue system without priority discipline, considered that equipment was subjected to imperfect repairs modeled by Generalized Renewal Process (GRP), a more general and flexible assumption.

As the market opens up for the entry of new firms, third-party agents can perform maintenance services with a satisfactory level of service, the bargain power of the customer increases, and he becomes capable to negotiate terms and clauses of the contract. Therefore, Principal-Agent framework allows the principal to choose the optimal incentives to design an optimal contract menu.

Santana et al. [10] were the first authors to use principal-agent game in the field of equipment reliability when modeling an MSC under the condition of information asymmetry. The optimal pricing and penalty clauses maximize principal’s utility. The authors considered that there are two types of agents in the market, with two different repair rates. and. These two profiles were known by the manufacturer who, however, was not able to discriminate the type of agent he was dealing directly with. The optimal contract guaranteed that efficient firms had no incentive (utility increase) to choose contracts designed for inefficient firms. However, despite designing an efficient agent type discrimination mechanism this paper still simplified for practical situations where exist a continuum of agent types. Taking this into account, the present dissertation intends to design a menu containing several types of contracts that can approximate the model to practical situations with three or more types of agents.

With regard to the reliability analysis of the systems, it is of great importance to make correct assumptions regarding the state of the system after a corrective maintenance. In practical terms, maintenance actions typically return the equipment to an intermediate condition between perfect and minimal repairs, which is called imperfect repair, situation modeled by Generalized Renewal Process (GRP), a more flexible mathematical model based on the concept of virtual age (Kijima & Sumita [11]; Wang & Pham [12]). The study of of warranty, maintenance outsourcing and MSC’s was done by Kim et al. [13], Bouguerra et al. [14], Husniah et al. [15], Huang et al. [16], andDarghouth et al. [2]. However, as [8] these authors considered that the system were perfect/minimally repaired.

As well as done by [9] that joined Stackelberg game and generalized renewal process (GRP) to model imperfect repairs, we intend to join PA approach with GRP to design a mechanism of auto selection of the service provider. The methodology developed by [17] will be applied to onbtain the Maximum Likelihood Estimators (MLE) for GRP parameters that describes the failure-repair process of the analyzed system, and use Discrete Event Simulation (DES) in order to obtain reliable estimates of the future behavior of an equipment and to support the MSC decision. Finally, the Particle Swarm Optimization (PSO) algorithm will be used as a tool to solve the non-linear decision problem formulated.

Differently from the above-mentioned works, Maintenance agents in the market has different repair rates that are governed by a cumulative distribution function , on a bounded interval . Fact that characterizes a situation of asymmetric information. Thus, the contracting firm has the challenge to design a contract that provides agents with the best incentives (pricing and penalties) for them to self-select. This paper unfolds as follows: in section 2 we describe the PA model, objective function and constraints of participation and incentive compatibility. In section 3 we present the flowchart with the resume of the tasks taken to solve the model. Section 4 we illustrate the applicability of the model with an application example, results and discussion. Finally, in section 5 we conclude remarks.

1. **MODEL DESCRIPTION**
   1. **Notation**

: average agent’s operational cost to perform a repair;

: type of the agent or complexity of the MSC;

: number of customers served by the agent;

: price of the MSC charged to an agent type ;

: probability associated to find in the market a firm belonging to the -th interval;

: mean revenue per hour;

: base warranty period;

: duration of MSC (after the end of the base warranty coverage);

simulation time horizon

: total time of operation of the system;

: total overtime for the system;

: principal’s utility associated to a wealth w;

: principal’s expected profit;

: time between the repair and failure;

time between the last failure and T;

: time to return equipment to operational state;

: time between failure and completion of repair the equipment after the failure;

: agents expected profit;

α: scale parameter of the Weibull distribution;

β: shape parameter of the Weibull distribution;

: penalty clause per hour to punish for delays in repairing equipment (decision variable);

δ: risk aversion parameter for the principal;

: repair rate of the agent type (exponentially distributed);

τ: agent's time threshold to return a failed device to an operational state without incurring penalty.

* 1. **Game formulation**

One firm called as the principal owns a complex equipment and plans to outsource maintenance actions to a third party (agent). When operational, the equipment returns an average revenue of per time unit. The firm is risk averse, its payoff is given by an exponential utility function (1) where represents the associated wealth and () is the degree of risk aversion.

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

When parameter decreases and tends to zero, the expected value of the principal's utility tends to its expected profit , which means that the principal is risk neutral. As grows, risk aversion level increases. After the expiration of the base warranty, due to predictability of total maintenance costs, the principal wants to design a fixed price menu of MSC. He will offer a menu of contracts with options. In each option, he will pay a fixed price to the agent to repair all failed units over at no additional cost. If a failed device is not returned to operational state within a period after a failure occurs, the OEM is charged a penalty per hour . The agent, in turn have two possible actions, he must decide whether to accept or not accept the contract. It is assumed that agents are risk neutral and so, the option will be chosen if agent’s expected profit is positive. The opposite will happen if the expected profit is negative.

However, the principal does not know about agent’s operational repair rate. Maintenance agents in the market are considered heterogeneous regarding their rates of repair which are distributed by a cumulative distribution function , on a bounded interval . The rapidity of the execution of the maintenance task is agent’s private knowledge, which characterizes a scenario of asymmetric information.

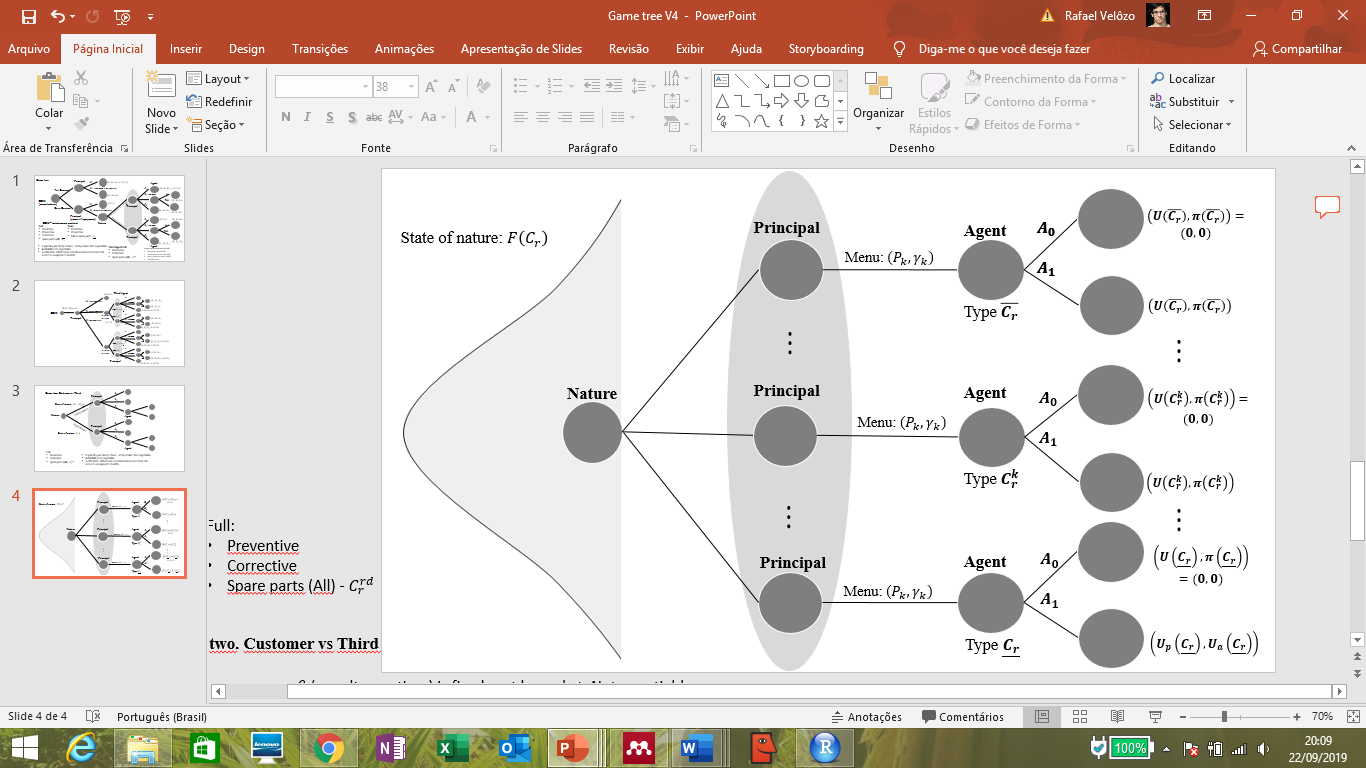
The time between the repair and failure is and the time between the start and completion of the failure of equipment and the completion of its respective repair is . If exceeds a threshold (the same for all agent types) a penalty will be incurred, proportional to the delay in the returning equipment to an operational state with structure , under the condition of (. This delay is called overtime and is denoted as ; , i.e., is the time between the instant of last failure and . Principal’s profit when an agent of type *k* performs maintenancewith rate is given by:

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

Where: ; and is the total number of failures of the device during the period of the contract with repairs executed by an agent with rate . All agents have the same mean cost of repair, and they only will sign the contract if their economic profit is greater than or equal to zero, covering its opportunity cost (IR constraint). The expected profits of a type *k* agent are given in (13):

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

The principal seeks to maximize his expected utility and intends to choose optimal pricing and penalty clauses menu for each agent , i.e., with the best incentives to be able to discriminate their profile and extract the maximum of agent’s utility (information rent). The agent, in turn, intends to maximize profit. At first, the principal must to decide the complexity of the contract, that means to define the number of contracts to be offered. Then, the agent chooses the contract that reflects his profile *k*. The chosen criteria for stablishing the partitions was dividing into *K* subintervals with equal probabilities. For example, for a menu with *K* = 4, the agents would be grouped in 4 subintervals, each with probability 25%. The described model can be visualized in Figure 1 on its extensive form, as a sequential game tree, containing all decisions and payoffs for both players.



*Figure 1- MSC design game tree. Source: the authors.*

At first, the nature plays. Afterwards, the principal takes an action by deciding which contract will be offered for each type of agent. At the moment of the proposal, the principal does not know with which type of agent he is playing. This fact is represented by the information set (ellipse in Figure 1) with *K* elements, which means that the principal does not know what nature's "move" was. Then, the agent defines whether to accept or decline the contract. As shown in Figure 2, the contract is accepted if the agent takes action , it is executed and the expected payoffs are obtained. Otherwise there is no agreement (). Here we represent nature as a player to make it easier to understand the dynamics of the game, even though it is not a player. and are the lower and the upper limits of the truncated distribution .

The agreement will be designed under information asymmetry since is not observable by the principal. This advantageous information can lead the agent to not behave according to the principal’s interests which characterizes a moral hazard situation. To avoid moral hazard, the principal will build a contract that prevents more efficient firms from taking a contract designed for less efficient firms, maximizing principal’s payoff and extracting maximum of agent’s surplus.

**2.3 Assumptions**

1. Equipment is repairable and subject to imperfect repair;
2. The equipment’s failures are critical. Moreover, the agent carries out just corrective maintenance interventions;
3. The time to first failure follows a Weibull distribution and the subsequent failures are governed by a Weibull-GRP;
4. Times to repair follow an exponential distribution with stochastic parameter
5. Both the principal and the agent have perfect information about the possible actions of each other;
6. The number M of devices maintained by the agent is revealed to the principal,
7. If there are more failed units than the number of servers, a queue following a FCFS is generated. This formulation describes a queuing system with finite population M.
8. Imperfect competition market structure. Multiple firms exist with the same capability to perform imperfect repairs with the same quality, so, the contract can be offered to these firms..

Assumption (VIII) enables the principal to offer the contract to other firms. If the agent’s expected payoff obtained by signing the contract covers agent's opportunity cost, (participation restriction) and if those firms have capacity to take another equipment, this means that if the number of equipment under the responsibility of the maintenance firm is below its optimum capacity (, as seen in Santana et al., (2018), the firm will close the agreement.

**2.4 Decision Problem**

Given the profits in equations (2-3), the principal maximizes the expected utility, given in (14), subject to the incentive compatibility and participation constraints, given by (5) and (6), respectively, which leads to the following non-linear programming problem:

|  |  |  |
| --- | --- | --- |
|  |  | (4) |
| s.t. |  | (5) |
|  |  | (6) |
|  |  | (7) |

Where the constraint in equation (5) is of IC (that prevent efficient firms to sign contracts designed to inefficient firms). IR’s constraint is given in equation (6) and ensure that the expected agent’s economic profit is non-negative, this constraint ensures that the agent will obtain by accepting the contract at least its opportunity cost. Note that as k increase the number of decision variables increase in order 2k and the number of constraints increase in order k². The greater the complexity of the contract, the more limited the search space becomes. This represent a more difficulty to find feasible solutions.

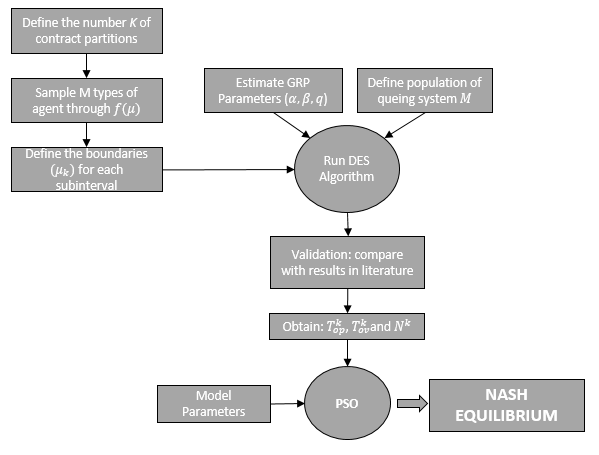
The expectation of the exponential in the objective function is a real-valued [random variable](https://en.wikipedia.org/wiki/Random_variable) that can be approximated through the approximation of Euller’s number by the Maclaurin series.With some algebraic work, the objective function can be rewritten in eq (8):

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

1. **SOLUTION**

The principal must set the desired number of contract options *K*, and group the agents in regions of the interval with the same probability of occurrence. Due to the flexibility to deal with different truncated distributions, we will resort to simulation to determine the boundaries of each interval. After defining the values of that define each interval, we will incorporate these repair rates into the DES model. Through simulation we will reproduce a queuing system, with limited population size M and an GRP arrival process (whose parameters were estimated by the maximum likelihood method). The needed estimates of the expected values of the random variables present in the decision problem will be obtained by DES. Finally, these estimates added to the other model parameters are used as input for a PSO algorithm to search the Nash equilibrium of the PA game.

A summary of the steps to solve the game are contained in the flowchart of Figure 2.



**NASH EQUILIBRIUM**

*Figure 2 - Steps for Nash Equilibrium*

1. **RESULTS AND DISCUSSION**

**4.1 Application Example**

As a demonstration of the applicability of PA model, an application example is shown in this section using a failure database of an angiography device, a medical equipment who supports the treatment and diagnosis of cardiovascular diseases. The angiography provides the visualization of arteries based on x-rays, and the injection of an iodine contrast material into blood vessels through a catheter. An angiogram provides anatomical information about blood vessels [18]. Through the flow of the contrast fluid, the doctor can identify obstructions and narrowing, and proceed with treatment [9].

It was analyzed 38 times between critical failures from an angiography database. It was considered that the device was subject to imperfect repairs, with this assumptions was obtained the MLEs for the GRP parameters by using the procedure described in Yañez et al [19]. The MLE’s obtained were h, , and . These parameters will be used as inputs for the application example.

The angiography device was bought at the cost of = $1,476 (10³), at the time of the purchasing, = 0, the principal will offer a menu of contract that will be implemented at the moment of the expiration of the base warranty (tied to the sale process). During the basic warranty period, the principal is fully compensated by the OEM for loss of revenue caused by downtime due to equipment repairs. We consider that the device is new and, as an assumption, all other devices served by the agent are also new. The duration of the base warranty is one year ( = 8,760 hours). The time horizon projected to be covered by the MSC’s is the second year of equipment operation from hours to hours. The parameters in table 1 were used to illustrate the applicability of the model:

*Table 1 – Parameters for the application example.*

|  |  |
| --- | --- |
| Revenue per time () | $ 0.094 (10³) h-1 |
| Mean repair rate () | 0.2 h-1 |
| Hospital’s risk aversion () | 0.0001 |
| Mean cost of repair | 5.4(10³) |
| Period of simulation () | 1 year = 8,760 h |
| Maximal time to repair the equipment under EW () | 8 h |

Then, after the determination of the intervals, uniting to the input of GRP parameters and the values of Table 1 into the DES algorithm we will obtain the expected values for the second year of operation from the machine, needed in the objective function. Additionally, the fact that the principal is fully compensated for loss of revenue in the first year (), the objective (8) function can be rearranged to:

|  |  |  |
| --- | --- | --- |
|  |  | (9) |

We will use the output of DES algorithm and the values in Table 1 to feed a PSO algorithm to search the optimal result for Eq. (9). The decision variables are components of the vector [], with dimension 2. The number of particles chosen for optimization were . The lower search bounds all initialized as zero for each decision variable and the values 200 and 20 (in order 10³$) were selected respectively for and as the upper bounds, for .

**4.2 Base results for**

To calculate the intervals for a contract complexity we assume that the population of agents is distributed by a truncated normal distribution with a lower bound 0.125 and an upper bound with mean = 0.2 and standard deviation = 0.08. By sampling a large number of types of agents through the generating function *truncnorm* contained in *SciPy* module in Python programming languagewe generated a curve by DES that approximate the theoretical distribution. Thenwe defined 2 regions [0.125, 0.217[ and [0.217, 0.4]. 0.217 is the median of the population and define these two regions (, that pools agents with costs of repair between [0.125, 0.217[ and [0.217, 0.4] respectively. The probability that the principal is negotiating with an agent from region is the same as he is negotiating with an agent from region , . The two lower limits will be chosen for optimization. and will be used as input of the DES algorithm in order to obtain model’s random variables estimates. The procedure given in 5 were executed using replications.

For , the number of random variables are twelve and the found expectations for = 0.125 were (rounding to two decimal places): 12.45, 8552.91, E[ = 127.65, = 73158104.86, = 20116.45, = 1087193.08.

And for = 0.217 these values were: = 12.63, = 8676.06, E[ = 25.15, = 75274998.06, = 952.56, = 217728.38.

As expected, the slower agent has higher overtime and lower availability. As its repair is slower, the equipment spends more time unavailable due to maintenance or waiting to be served in queue.Finally, we are now ready to run the PSO algorithm to obtain the Nash Equilibrium of the game under asymmetric information. In order to attest to the effectiveness of the decision obtained through the principal-agent game, we also optimized the contract under the assumption of complete information, called as First-Best (FB) contract. The principal’s optimal decision - pricing, penalties and expected utilities for each case are given in Table 2, we highlight the results found by PA model in grey.Values are 10³.

*Table 2 - Optimal menu offered by the principal with k = 2 contract options*

|  |  |  |  |
| --- | --- | --- | --- |
| **PA model ($)** | | **First-best optimal menu ()($)** | |
|  | 68.39 |  | 80.39 |
|  | 132.20 |  | 69.90 |
|  | 0.01 |  | 0.10 |
|  | 2.54 |  | 0.07 |
|  | 1,442.898 |  | 1,442.945 |

From the optimal utility obtained by the principal-agent game, we obtained the expected utility generated in the first year, an amount of 773.294. This value is a certain gain obtained by the fact that the principal is completely compensated for the loss of revenue from OEM’s base warranty. For the principal sign the maintenance contract for the second

By comparing the First-Best contract with the principal-agent menu, in terms of expected value, we observe that the obtained utility in the first-best contract is slightly superior then the principal-agent menu, but the difference is almost negligible both in absolute and relative terms ($0.047(10³) and 0.00326% respectively). However, in practical transactions, the repair rate is an information owned by the agent only, and if the First-Best menu were offered to the agents, the principal would fail in his attempt to propose a self-selection mechanism, and the type 1 (with rate agent would choose the contract assigned to a type 2 (with rate) as we can see in table 3. Thus, we conclude that offering the same FB menu in an information asymmetry setting is not an efficient strategy, it gives the agent a large amount of informational rent. This can be seen in Table 5, where agents are motivated to choose the contract devised to other types of agent to get positive payoff.

*Table 3 - Nash equilibrium leads to agent's self-selection*

|  |  |  |  |
| --- | --- | --- | --- |
| **Class of agent** | **Type of contract** | **Profit (PA model) ($)** | **First-Best contract**  **($)** |
|  |  | 0.00 | 0.00 |
|  |  | -259.85 | -5.97 |
|  |  | -0.04 | 9.60 |
|  |  | 0.00 | 0.00 |

Note in the last column that the agent of type 2 is motivated to choose the contract directed to type 1 agents so that he would get positive profit of $9.60(10³). This is the type information rent, value that increases as approaches 0.4 (agent becomes faster). The result found by our model encourages a self-selection mechanism for all possible scenarios of the agent lying about his type, he will have negative profit. Also, the PA model allows the principal to extract all information rent, leaving the agent with no trading surplus.

1. **CONCLUSION**

In this paper we designed a contract that provides to maintenance agents the best incentives (pricing and penalties) for them to self-select. This developed PA approach changed the MSC’s perspective for the owner of the equipment viewpoint, in a setting where there is greater competition between service providers than it happens at the monopolistic scenario seen in the Stackelberg model [9].

Additionally, an application example was presented with real failure data of a clinical device to determine the optimal strategies for each player and demonstrate applicability of the model. Although we designed an efficient agent type discrimination mechanism, for practical situations where exist a continuum of agent types this work still simplified because we had a few the number of contract partitions.

A difficulty faced for problems with complexity greater than 3 was to find viable solutions for initializing the PSO algorithm. Therefore it was not possible to design a menu containing a M-large types of contracts to discretize and approximate the continuous Principal-Agent problem and maximize the trading efficiency in practical situations.

Further researches can also consider:

* That the number of equipment under agent’s responsibility is not revealed to de principal (agent’s private information).
* Preventive maintenance policies can be incorporated into the model, evaluated and optimal policies determined.
* Historical failure data can be updated with observations obtained during the first year of equipment operation in a Bayesian approach.
* Incorporation of a heterogeneous risk averse maintenance agent’s market, with different risk profiles of agents, in an agent-based game approach

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