Studies on the Optimization of Test Intervals for Safety Systems Considering Damage Induced by Tests

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ABSTRACT

Testing of safety systems is essential to ensure that their probability of failure on demand (PFD) is maintained below acceptable limits throughout their lifetime. It is commonly assumed that more frequent testing and a shorter interval between tests imply a lower PFD, because test results provide information about the functional status of components. Nevertheless, it is also well known that tests may induce stresses in the tested components which tend to accumulate with time. The traditional models for evaluation of PFD with constant failure rates do not capture the damaging effect of the tests.

In this paper, we derive optimal testing policies under two different scheduling schemes: (i) periodic testing with equal intervals between tests, and; (ii) adaptive scheduling, where test intervals are allowed to vary throughout the operational cycle. We show that adaptive test scheduling with long initial test intervals that gradually decrease as the component accumulates damage give a lower average PFD than a periodic testing with constant intervals for the same total number of tests.

1. INTRODUCTION

Testing of safety systems is essential to ensure that their probability of failure on demand (PFD) are maintained below acceptable limits throughout their lifetime (see IEC-61508 (2010)). It is commonly assumed that more frequent testing, with shorter intervals between tests, implies a lower PFD, because test results provide information about the functional status of components. Nevertheless, it is also well known that tests may induce stresses in the tested components (BSEE, 2015). In fact, some tests may induce levels of stress that over time will accumulate and may lead to significant damage to the tested components. This means that, for a fixed operational interval, increasing the number of tests, or, equivalently, reducing the intervals between tests, may actually increase the average PFD of the safety system within the operational cycle. A practical case that called our attention to this problem was the difference of opinion between the US Bureau of Safety and Environmental Enforcement (BSEE) and the offshore industry regarding the frequency of pressure testing of subsea BOPs during the development of the Well Control Rule (BSEE, 2015). While the industry proposed that BOP pressure tests be performed every 21 days, as indicated in API 53 (API, 2018), BSEE responded that:

> *"BSEE is not aware, however, of any new data that justifies increasing the BOP pressure testing interval for all BOPs from 14 days to 21 days."*

Considering an operational cycle of 5 years for a BOP (recertification at 5-year period) established in the Well Control Rule (BSEE, 2015), the variation in the number of pressure tests to be performed in that period goes from 130 to 87, respectively for 14 to 21 days pressure test intervals. By sticking to the shorter interval, BSEE is assuming that the shorter interval between pressure tests ensure a lower PFD for the BOP. But considering the damage induced by the tests, is this policy the best one, or are companies testing their BOPs too much?

In Reference (BSEE, 2015) BSEE states that the estimated operational costs to the companies due to the frequency of pressure testing jumps from USD150 million for the 21-day policy to USD400 million for the 14-day one, a very significant difference by any account. But in this paper, we are only concerned with the safety aspects of the problem. It is also important to say that the 14- day interval is also adopted as a requirement by other regulators around the world (e.g., Norway PSA (NORSOK, 2013)).

Optimization of test, inspection and repair scheduling is a topic that has been studied extensively in different contexts, both analytically and numerically. Kim et al. (1994) evaluated optimal test frequency based on the trade-off between the beneficial and adverse effects of testing, considering both time related and test related aging of components. Kaio et al. (1994) considers optimal non-periodic test scheduling when tests induce immediate failure with a prescribed probability. Lapa et al. (1999, 2000) have used a genetic algorithm approach to search for time points when it is optimal to perform testing, inspection or intervention, also considering aging of standby components (Lapa et al., 2002). The current paper complements the above studies, by providing analytical results for both periodic and non-periodic test policies, focusing specifically on the gradual damage induced by tests on the tested components. We show that the existence of an optimum periodic test interval depends on the nature of the degradation process (i.e. how fast the failure rate increases as a function of the number of tests). We also show that there always exists an optimal non-periodic strategy such that the benefit of testing outweighs the detrimental damage effect.

2. THE MODELS

The time-dependent probability of failure on demand - PFD(t) - is the probability that the safety system is in a failed state when a hazardous demand occurs at time t. We are here focusing on the undetected failures during operations, and the purpose of the tests is to uncover the actual state of the safety system, and thus to reduce the PFD. The PFD is therefore equal to the unavailability of the safety system. The undetected failures are not repaired or rectified when they occur (they cannot, since they are not detected); accordingly, PFD(t) is equal to the probability that a failure has occurred before t. The general equation for PFD(t) can then be written as:

$$PFD(t+dt) = PFD(t) + [1 - PFD(t)]\,\lambda(t)dt \tag{1}$$

This equation states that the probability that the safety system has failed at t + dt is equal to the probability of it being failed at t, plus the probability of it not being failed at t and then failing within the next interval dt. In Eq. (1), $\lambda(t)$ is the undetected failure rate. Eq. (1) can be expressed as the differential equation:

$$\frac{PFD(t)}{1 - PFD(t)} = \lambda(t)dt, \tag{2}$$

which can easily be solved by direct integration from the start of a test at $t = t_i$ to time t:

$$PFD(t) = 1 - [1 - PFD(t_i)])e^{-\int_{t_i}^{t} \lambda(t)dt}. \tag{3}$$

We now consider a component that starts out as new (i.e., working condition) at time $t_0 = 0$ and is subject to N functional test at subsequent time points $\{t_1, t_2, \dots, t_N\}$. We let $t_{N+1} = T$ be the total operational cycle of the component and assume that the failure rate is piecewise constant between tests. This means that $\lambda(t) = \lambda_i$ where i means the interval after i tests. Furthermore, we will assume that tests are perfect (i.e. all failures are detected) and that any failed component is repaired to the condition it was in the interval before the failure was detected (i.e. as-good-as-old repair). The latter is a conservative assumption. This means that $PFD(t_i) = 0$. To simplify notations, we will let f(t) = PFD(t) in the following. The probability of failure on demand at time $t \in [0; T]$ is then given by

$$f(t) = \begin{cases} 1 - e^{-\lambda_0 \cdot t} & 0 < t \le t_1 \\ 1 - e^{-\lambda_1 \cdot (t - t_1)} & t_1 < t \le t_2 \\ \dots & \\ 1 - e^{-\lambda_N \cdot (t - t_N)} & t_N < t \le T. \end{cases} \tag{4}$$

With_$\Delta t_i = t_{i+1} - t_i$ as the duration between tests, the average PFD in interval i becomes

$$\begin{aligned} \bar{f}_i &= \frac{1}{\Delta t_i} \int_{t_i}^{t_{i+1}} 1 - e^{-\lambda_i (t - t_i)} dt \\ &= 1 - \frac{1 - e^{-\lambda_i \Delta t_i}}{\lambda_i \Delta t_i}. \end{aligned} \tag{5}$$

The average PFD over the interval [0; T] can be computed using Eq. (4):

$$\begin{aligned} \bar{f}(N) &= \frac{1}{T} \int_0^T f(t)\, dt \\ &= \frac{1}{T} \sum_{i=0}^{N} \int_{t_i}^{t_{i+1}} 1 - e^{-\lambda_i \cdot (t - t_i)} dt \\ &= 1 - \sum_{i=0}^{N} \frac{1 - e^{-\lambda_i \cdot \Delta t_i}}{\lambda_i T}. \end{aligned} \tag{6}$$

We will use this formula repeatedly throughout this paper.

2.1 Equal Test Intervals

We first consider the case where tests are performed at constant intervals of length $\Delta t = T/(N+1)$, i.e. at times $t_i = iT/(N + 1)$, any $i \in \{1, 2, \dots, N\}$. Then Eq. (6) becomes

$$\bar{f}^{\mathrm{eq}}(N) = 1 - \sum_{i=0}^{N} \frac{1 - e^{-\lambda_i T/(N+1)}}{\lambda_i T}. \tag{7}$$

In the limit of small failure rates, i.e. where $\lambda_i \Delta t << 1$, Eq.(7) can be approximated to lowest order by

$$\begin{aligned} \bar{f}^{\mathrm{eq}}(N) &\approx 1 - \sum_{i=0}^{N} \frac{\Delta t \lambda_i - (\Delta t \lambda_i)^2/2}{\lambda_i T} \\ &= \frac{\Delta t^2}{2T} \sum_{i=0}^{N} \lambda_i \\ &= \frac{T}{2(N+1)^2} \sum_{i=0}^{N} \lambda_i \\ &= \frac{T}{2(N+1)} \langle \lambda \rangle_{N,\mathrm{arithmetic}}, \end{aligned} \tag{8}$$

where $\langle \lambda \rangle_{N,\mathrm{arithmetic}}$ denotes the arithmetic mean of the failure rates over the failed states.

2.2 Optimal Test Intervals

We define optimal test intervals by the set $\{t_1, t_2, \ldots, t_N\}$ that minimizes the average PFD over the operational cycle [0, T]. This amounts to minimizing Eq.(6), which occurs when

$$\frac{\partial}{\partial t_i}\bar{f}(N) = 0 \quad \forall i \in \{1, 2, \ldots, N\}, \tag{9}$$

giving the set of equations

$$\{e^{-\lambda_{i-1}\cdot\Delta t_i} - e^{-\lambda_i\cdot\Delta t_i} = 0\}_{i=1}^{N}, \tag{10}$$

or equivalently

$$\{\lambda_{i-1}\cdot\Delta t_{i-1} = \lambda_i \cdot \Delta t_i. \tag{11}$$

At the optimal test intervals, we therefore conclude that $\lambda_i \Delta t_i$ = constant.

Since $\sum_{i=0}^{N} \Delta t_i = T$ we see immediately that $T = constant \sum_{i=0}^{N} 1/\lambda_i$ so that

$$\Delta t_i = \frac{constant}{\lambda_i} = \frac{T}{\lambda_i C_N}, \tag{12}$$

where $C_N = \sum_{i=0}^{N} \frac{1}{\lambda_i}$. The corresponding average PFD is given by

$$\begin{aligned} \bar{f}^{\text{opt}}(N) &= 1 - \sum_{i=0}^{N} \frac{1 - e^{-\lambda_i \Delta t_i}}{\lambda_i T} \\ &= 1 - \frac{C_N(1 - e^{-T/C_N})}{T}. \end{aligned} \tag{13}$$

In the limit of small failure rates where $\lambda_i \Delta t \ll 1$, we see that C_N becomes large, hence Eq. (13) can be approximated to lowest order by

$$\begin{aligned} \bar{f}^{\text{opt}}(N) &= 1 - \frac{C_N(1 - e^{-T/C_N})}{T} \\ &\approx \frac{T}{2C_N} \\ &= \frac{T}{2(N+1)} \langle\lambda\rangle_{N,\text{harmonic}}, \end{aligned} \tag{14}$$

where $\langle\lambda\rangle_{N,\text{harmonic}} = (N+1)/C_N$ denotes the harmonic mean over the failure rates.
Note the similarity in form between Eq.(8) and Eq.(14), where the only difference is the type of average over the failure rates.

3. RESULTS

3.1 Effect of test damage with periodic testing

In the case where there is no damage from the test, the failure rate $\lambda_i = \lambda$ is constant. In this well-known situation the average PFD follows directly from Eq.(7) as

$$\bar{f}^{\rm eq}(N) = 1 - \sum_{i=0}^{N} \frac{1 - e^{-\lambda T/(N+1)}}{\lambda T}$$
$$= 1 - \frac{N+1}{\lambda T}\left(1 - e^{-\lambda T/(N+1)}\right) \quad (15)$$
$$= \phi\left(\frac{\lambda T}{N+1}\right),$$

where we have introduced the function

$$\phi(x) = 1 - \frac{1}{x}\left(1 - e^{-x}\right) \quad (16)$$

Since ϕ is monotonically increasing with x we see immediately that increasing the number of tests N will always reduce the probability of failure on demand.

We now turn to the case where the tests induce damage to the system, addressing the question on test frequency for BOPs as a case-in-point. The average PFD over the total test interval Eq.(7) now becomes.

$$\bar{f}^{\rm eq}(N) = 1 - \sum_{i=0}^{N} \frac{1 - e^{-\lambda_i T/(N+1)}}{\lambda_i T}. \quad (17)$$

We note that the asymptotic behaviour $\bar{f}^{\rm eq}(N)$ of as N → ∞, depends on the asymptotic behaviour of λ_N as N → ∞, i.e.:

$$\lim_{N\to\infty} \bar{f}^{\rm eq}(N) = \begin{cases} 0 & \text{if } \lim_{N\to\infty} \frac{\lambda_N}{N} = 0 \\ \phi(\alpha T) & \text{if } \lim_{N\to\infty} \frac{\lambda_N}{N} = \alpha, \\ 1 & \text{if } \lim_{N\to\infty} \frac{\lambda_N}{N} = \infty \end{cases} \quad (18)$$

This means that, depending on how much the tests affect the failure rate, more tests may either improve or reduce the average PFD. In the special case where the failure rate increases linearly with the number of tests, the average PFD tends towards a constant and finite value $\phi(\alpha T)$.
To be specific we will consider two degradation models to illustrate the point above. In the Additive Test-Step Varying (ATSV) model (Oliveira et al., 2016) the failure rate is increased by a fixed amount at every test and remains constant in-between tests. The model can, loosely speaking, be said to represent ”linear damage”, as the damage from any test is the same. The failure rate after test i of the ATSV models is

$$\lambda_i = (1 + ai)\lambda_0. \quad (19)$$

Here a is a constant parameter assumed positive. In this model, additional tests will in general reduce the PFD. From Eq. (18) we see that the PFD will ultimately approach the finite limit $\phi(aT\lambda_0)$ for large number of tests and cannot be reduced any further by additional testing.
The second model we will consider is the Multiplicative Test-Step Varying (MTSV) model (L. F. Oliveira and J. Domingues, 2016; G. A. Vale, 2018) where the damage from the test is not additive like in the ATSV model, but rather multiplicative. The model can, loosely speaking, be said to represent ”compound damage”, as the damage from an early test is much smaller than damage from a later test. The failure rate after test i of the MTSV models is

$$\lambda_i = (1+a)^i \lambda_0. \qquad (20)$$

In this model, additional tests will not necessarily reduce the PFD. Initially, the PFD will be reduced by more testing, as the test damage is initially small. However, after sufficiently many tests, the damage has accumulated into a sufficiently large failure rate so that further testing will increase the PFD. From Eq. (18) we see that the PFD upon further tests will approach 1, i.e. a certain failure.

A similar curve as that of the MTSV model, with a minimal value of the average PFD, has also been found in the work of Srivastav et al. (2018). The minimal value of the PFD for the MTSV model can be determined for the MTSV model. To illustrate the occurrence of the minimum, we use, for simplicity, the linear approximation from Eq.(5), which for the MTSV model reads

$$\begin{aligned} \bar{f}^{\text{equal}} &\approx \frac{T}{2(N+1)^2}\sum_{i=0}^{N}(1+a)^i\lambda_0 \\ &= \frac{T\lambda_0\left((1+a)^{N+1}-1\right)}{2a(N+1)^2}. \end{aligned} \qquad (21)$$

The optimum occurs whenever d = $df^{equal}/dN = 0$, which for large N can be calculated from the first few terms in the expansion as

$$N^{\max} = \frac{2}{\ln(1+a)}\left(1-e^{-2}\right)-1, \qquad (22)$$

independent of both λ_0 and T. Hence, any additional testing beyond this maximum number of tests will then increase the PFD. As a check on the linear approximation, we see that at the minimum $\lambda_N^{max} = e^2\,\lambda 0$, so the linearisation will in general still be valid at the minimum PFD. In Figure 1 we have plotted the PFD for both the ATSV and MTSV model for comparison. We see clearly that in the ATSV model the PFD is declining towards a constant value $\phi(aT\lambda_0)$ as the number of tests grow. On the contrary, the MTSV model displays a minimum for a given number of tests and then starts to rise again upon further testing. The maximum number of tests before the PFD starts to rise again is well approximated by Eq. (22).

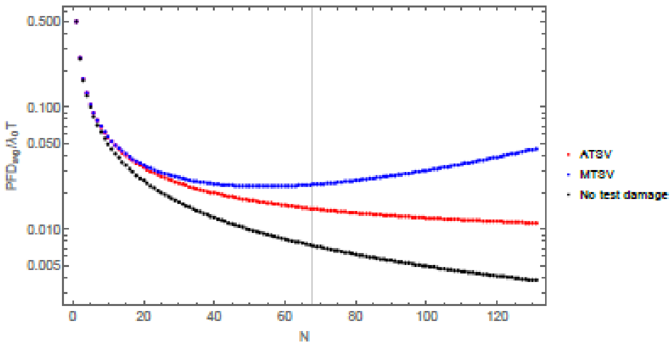

Fig. 1. Plot of the PFD for the additive model (ATSV) and multiplicative model (MTSV) for damage parameter $a = 0.03$. The additive model approaches a finite value for large number of tests, while the multiplicative model displays a clear minimum, corresponding to a maximum number of tests, that is given approximately by Eq. (22) for $N^{\max} = 57$, which is indicated with a grey line. For comparison, the case with no test induced damage is shown in black.

3.2 Effect of optimal test intervals

We now consider the case of optimal test intervals as determined in Sec 2.2. We note from Eq.(12) that for constant failure rates, the optimal test interval is also constant, with $\Delta t_i = \Delta t = T/(N + 1)$. However, if the failure rate is affected by the tests, the optimal test schedule is not equipartitioned. Stated differently, periodic testing is not optimal when there is damage caused by the tests. In Figures 2 and 3 we have plotted the average PFD for both the equipartitioned case and the optimal test interval case, for both the ATSV and MTSV models respectively. We immediately see two distinct features: (i) The PFD is always lower for the case with optimal spacing, and; (ii) the PFD is always reduced with more tests. The first feature is expected and can also easily be seen by comparing the two expressions in Eqs.(8) and (14) noting that we always have $\lambda_{N;harmonic} \leq \lambda_{N;arithmetic}$.

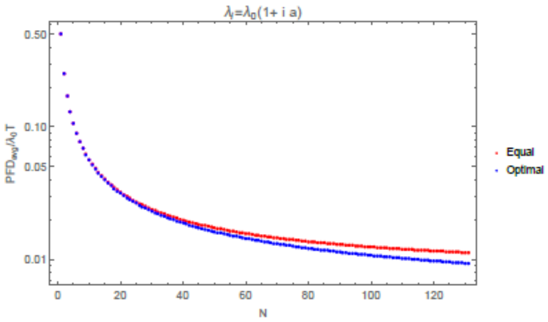

Fig. 2. Additive model. Note that testing more is always better, but the optimal placement of tests reduces the average PFD more for the same number of tests. Plotted for damage parameter $a = 0.03$.

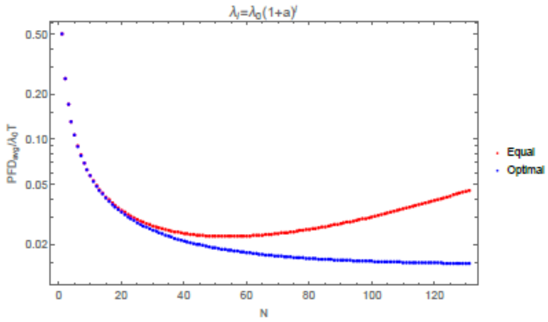

Fig. 3. Multiplicative model. Note that testing more is not always better if the test are spaced equally. however, with optimal placement of test, more testing always reduces the average PFD. Plotted for damage parameter $a = 0.03$.

The second feature is more surprising. To better understand this feature we note that at optimal test intervals the average PFD in interval i found in Eq.(2) turns out to be identical in all intervals:

$$\bar{f}_i^{\text{opt}} = 1 - \frac{C_N(1 - e^{-T/C_N})}{T}. \tag{23}$$

Furthermore, they are equal to the average PFD over the entire interval found in Eq.(10). So, for optimal test intervals, we have

$$\bar{f}^{\text{opt}}(N) = \bar{f}_i = 1 - \frac{C_N(1 - e^{-T/C_N})}{T}. \tag{24}$$

Since the interval PFD and the total interval PFD are identical, we understand easily why adding a test always will reduce the PFD: If you add a next interval Δt_{N+1} beyond T the total interval PFD will remain constant. If we now ”compress” these N +1 tests within the original T interval, all intervals will be shorter and all interval-PFD will be reduced. The overall PFD will consequently also be reduced. Moreover, we see that since C_N is a monotonically increasing function of N, the asymptotic behaviour of the PFD in Eq. (24) is

$$\lim_{N\to\infty} \bar{f}^{\text{opt}}(N) = 0. \tag{25}$$

Thus, unlike the case with equal test intervals, the average PFD will always decrease towards zero with more tests if the test times are selected optimally.

4. DISCUSSION

From the findings for the case with equipartitioned tests (periodic testing), we saw that increasing the number of tests did not necessarily reduce the PFD. Crucially, it depended on the damage process in question. Thus, having a good understanding on the stress mechanisms related to the tests becomes paramount in deciding on the test interval duration in the equipartitioned case. The latter is needed to inform the decision on an optimal test schedule for BOPs as presented in the introduction. The number of tests corresponding to a 2 week or 3 week test interval for a 5 year cycle for a BOP was 130 and 87, respectively. From the optimal number of tests in the multiplicative model, we see that this corresponds to damage parameter a = 0.023 and a = 0.015, respectively. These represent small damage effects, making the effect exhibited by the model more likely to represent real observable effects. The different features from the different damage models observed in the case of equal test intervals is not present in the optimal test interval.

In the latter case, the PFD is always reduced by additional tests, irrespective of the damage mechanism involved. This makes this test strategy more robust towards the physical features of the damage mechanisms.

The optimal test interval approach can easily be implemented in practical applications. In this optimal regime both the average and peak PFD in every interval is constant. This suggest a practical way to schedule test optimally during operations, when knowledge from previous test make us able to better determine the current failure rate of the actual components in use: After each test, simply schedule the next test when the PFD reaches the same PFD value as when the current test was performed.

We also note that in the optimal test interval strategy, the average PFD remains constant in each interval and thus throughout the entire campaign. From a safety perspective, this is in itself attractive as it avoids the traditional case where it is expected that failures occur more frequently as the campaign unfolds. A challenge with the optimal strategy is, however, that as the failure rate grows, the inspection interval gets shorter, and at some stage it becomes impractical to test too often. From Eq.(12) we can deduce that

$$\frac{\Delta t_i}{\Delta t_0} = \frac{\lambda_0}{\lambda_i}, \tag{26}$$

However, even in such cases it may be possible to find suboptimal test schedules that are practical to implement and give a lower PFD than obtained with constant test intervals.
This challenge occurs because the total number of tests is constant and we optimise the test intervals, resulting in shorter and shorter test intervals towards the end of the campaign. There is an alternative approach that circumvents this challenge. If we instead of keeping the number of tests constant, require the PFD to be the same when going from periodic to optimal schedule, we have instead

$$\bar{f}^{\mathrm{eq}}(N) = \bar{f}^{\mathrm{opt}}(N^{\mathrm{opt}}). \tag{27}$$

Here N^{opt} is in general smaller than N, as can be observed from Figures 2 and 3. To illustrate this, we use the linear approximations in Eq.(8) and Eq.(14), from which it then follows that

$$\frac{1}{N^{\mathrm{opt}}+1}\langle\lambda\rangle_{N^{\mathrm{opt}},\mathrm{harmonic}} = \frac{1}{N+1}\langle\lambda\rangle_{N,\mathrm{arithmetic}}. \tag{28}$$

From this one may easily calculate Nopt for a given N. We illustrate this in Figure 4 for the ATSV and MTSV degradation models. As an alternative to a periodic test schedule with N tests one may therefore use optimal test spacing with only Nopt tests and still keep the average PFD for the entire interval at exactly the same level.

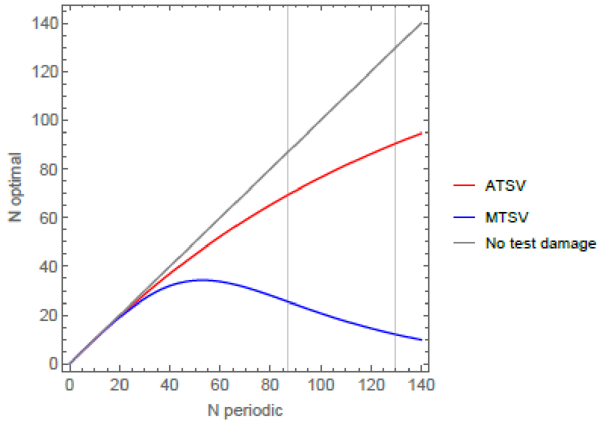

Fig. 4. Number of optimal tests needed to obtain same PFD as with a given number of periodic tests, plotted for ATSV and MTSV models, for damage parameter $a = 0.03$. The vertical lines indicate 87 and 130 tests, corresponding to 14 and 21 day inspection intervals over a 5 year period, respectively.

5. CONCLUSION

We have shown in this paper that tests may induce damage to the tested components that may outweigh their usual reduction of the average PFD. Using constant test intervals it is shown that, depending on the degradation mechanism/model, there may exist an optimal number of tests, i.e., a number of tests that results in the lowest value of the average PFD over the whole operational period. With the additive model (ATSV), where the degradation is linear in the number of tests, the degradation caused by the test is exactly compensated by the reduction of the test interval, leading to convergence of the average PFD at a finite nonzero value. More testing always reduces the PFD if the degradation mechanisms is slower than linear in the number of tests, and if the degradation is faster than linear (e.g. MTSV model) the detrimental effect of degradation will eventually dominate the positive effects of performing tests, such that an optimal number exists. However, we have also shown that the optimal way to test is not the use fixed periodic test intervals, but to adopt an adaptively changing interval which decreases as the failure rate increases due to test induced degradation. Using this optimal adaptive test scheduling, the average PFD always decreases when more tests are added within a fixed operational lifecycle of the equipment.

While this paper does not answer what the optimal test strategy is for a specific system such as a BOP, we conclude that understanding the impact of testing on component degradation is important for coming up with good test strategies.

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